Universal Approach to Polynomial Chaos Expansion for Stochastic Analysis of EM Field Propagation on Convex Obstacles in an UWB Channel

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***Abstract*— This paper presents a new universal approach for simulation of electromagnetic (EM) wave propagation in ultra-wideband (UWB) channel focusing on statistical analysis of EM field distribution. We present the new approach for the case of diffraction on convex obstacles. We base on a circular cylinder model of a convex obstacle and uniform theory of diffraction (UTD) which can be effectively used in an asymptotic prediction of an EM field scattered by convex obstacles. The polynomial chaos expansion is used in the statistical analysis. It is derived using the Hermite polynomial basis.**

***Index Terms*— UWB propagation, polynomial chaos, stochastic simulation.**

I. INTRODUCTION

UWB technology brought a lot of attention worldwide due to its advantages that can be implemented in communications and radar area. Nowadays the usual UWB propagation channel concerns indoor scenarios. These channels usually exhibits the presence of convex obstacles e.g. human body. For the proper analysis of UWB propagation it is essential to include an ultra-wideband interaction of EM wave with an obstacle. This interaction results in a signal distortion, which is practically not present in a narrow-band EM wave propagation. It is important to have predictive tools to obtain as accurate as possible field distribution in order to enable optimized implementation of an UWB transmission system. The simulators of EM wave propagation base on different models of propagation environment as empirical, statistical, site-specific, theoretical, etc. In this paper we deal with theoretical modeling of EM wave indoor propagation focusing on convex obstacles which can be static, e.g. rounded pillars or non-static, e.g. humans. The essential advantage of physical models is enabling of a detailed insight into an influence of physical phenomena and physical parameters of a channel on an observed EM field.

This paper provides the new universal approach to statistical analysis of a relationship between channel physical parameters and an EM field distribution. For the clarity of a presentation of the approach and space saving issues the work presented in this paper concentrates on analysis of a diffraction phenomenon on a convex obstacle modeled by a circular 2D cylinder. We use uniform theory of diffraction in our simulations. The propagation scenario of an UTD diffraction ray is presented in Fig. 1. Although numerical examples in the paper concern only a diffraction scenario our new approach presented in Section II can be easily adopted for e.g. reflection phenomenon on a convex obstacle.

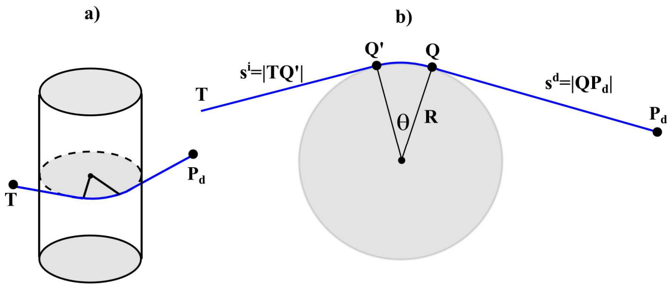

Fig. 1. 2D scenario of a diffraction (creeping) ray traveling along a circular cylinder – a) and cross-section of the cylinder – b).

In practical applications it is very important to include stochastic properties of physical parameters of propagation channel elements. Different numerical methods enable to include a stochastic behavior of a simulation problem. Among them we can mention Monte Carlo method, moment equations, perturbation techniques, polynomial chaos, etc [1]. The main goal of these techniques is to deliver more reliable simulation results dealing with an imprecision of given channel scenario parameters. In our analysis we use the polynomial chaos technique, which is very effective and provides the results in much less time than the Monte Carlo method. Polynomial chaos expansion has been introduced to computational problems in electromagnetic [2] and recently used in e.g. [3]. To our knowledge until now no study was focused on application of the polynomial chaos for an ultra-wideband propagation therein concerning convex obstacles. Furthermore the past results concerning this subject enable a direct derivation of a filed distribution only for one (assumed) value of mean and variance of simulation input parameters. If a mean

The presented work has been funded by the Polish Ministry of Science and Higher Education within the status activity task 2016 in “Development of methods for the analysis of propagation and signal processing as well as EMC issues”

or a standard deviation changes then new time consuming numerical calculations are required.

In this paper we present the universal approach to a polynomial chaos expansion where the coefficients of the derived expansion are the functions of a mean and a standard deviation of the input stochastic variables as well as the frequency. In order to obtain the universal expansion we apply the Hermite polynomial basis. We verify the exemplary simulation results obtained with our universal expansion by comparing it with the results of the Monte Carlo method.

The paper is organized as follows. In Section II we introduce our universal approach to a polynomial chaos expansion dedicated for effective stochastic simulations of EM wave propagation in an UWB channel. Section III gives some numerical examples that verify our new approach for the case of a diffraction on a convex obstacle modeled by a PEC 2D circular cylinder. We conclude the paper in Section IV.

II. THE UNIVERSAL POLYNOMIAL CHAOS EXPANSION

Polynomial chaos expansion enables to express a considered function of stochastic variables as a spectral expansion of a these uncertain variables. When the stochastic variables have a Gaussian distribution the strong convergence is obtained when an orthogonal basis of the Hermite polynomials is used. The Hermite polynomial chaos expansion of a transfer function $T(\omega_n,\xi)$ for a specific pulsation ω_n with respect to input stochastic variable ξ whose mean and standard deviation are μ_0 and σ_0, respectively can be calculated according to the formula:

$$T(\omega_n,\xi)=\sum_{k=0}^{\infty}a_{k,n}H_k\left(\frac{\xi-\mu_0}{\sigma_0}\right), \tag{1}$$

where coefficients of an expansion are calculated by:

$$a_{k,n}=\frac{1}{\gamma_k}\int_{-\infty}^{\infty}T(\omega_n,\xi)H_k\left(\frac{\xi-\mu_0}{\sigma_0}\right)\frac{\exp\left(\frac{-(\xi-\mu_0)^2}{2\sigma_0^{\,2}}\right)}{\sqrt{2\pi}\sigma_0}d\xi\ , \tag{2a}$$

or

$$a_{k,n}=\frac{1}{\gamma_k}\int_{-\infty}^{\infty}T(\omega_n,\sigma_0\xi+\mu_0)H_k(\xi)\frac{\exp\left(\frac{-\xi^2}{2}\right)}{\sqrt{2\pi}}d\xi\ , \tag{2b}$$

where n is the number of a frequency sample while:

$$\gamma_k=\int_{-\infty}^{\infty}H_k(\xi)\cdot H_k(\xi)\frac{\exp\left(\frac{-\xi^2}{2}\right)}{\sqrt{2\pi}}d\xi=k!\ \cdot \tag{3}$$

$H_k[.]$ is the Hermite polynomial of the kth order. The form of the series (1) results from the need to ensure its faster convergence by suitably choosing μ_0, σ_0. In the scenario of a diffraction (creeping) ray shown in Fig. 1, we can deal with three stochastic variables, i.e. $\xi=R$, $\xi=\theta$ and $\xi=L_d=s^is^d(s^i+s^d)^{-1}$. After finding the expansion coefficients of function $T(\omega_n,\xi)$, its mean and variance are defined by (4) and (5), respectively.

$$\mu_{T(n)}=a_{0,n}\ , \tag{4}$$

$$\sigma_{T(n)}=\sqrt{\sum_{k=1}^{\infty}k!\cdot a_{k,n}^{\ \ 2}}\ . \tag{5}$$

In order to find an ultra-wideband function of expansion coefficients corresponding to μ_0 and σ_0 the coefficients are calculated numerically for all the pulsation samples ω_n in the UWB frequency domain. Then we use the vector fitting algorithm [4] in order to obtain the frequency dependent expansion coefficients in terms of rational functions [4,5]. Afterwards the approximated transfer function can be easily transformed into the time-domain by using the inverse Laplace transform. As a result we obtain a spectral expansion of an impulse response which is expressed by a sum of exponential functions what allows an application of an effective calculation of a convolution with a given UWB signal.

The vector fitting algorithm is applied only once for each of the expansion coefficients. It is performed only for ξ with mean and standard deviation equal μ_0 and σ_0, respectively. In order to derive expansion coefficients for ξ with mean and standard deviation μ and σ, respectively, we proceed as follows.

The goal is to find a chaotic polynomial expansion given by (6) using known (tabulated) expansion coefficients (2) which was calculated with respect to variable ξ whose mean and standard deviation are μ_0 and σ_0, respectively.

$$T(\omega_n,\xi)=\sum_{m=0}^{\infty}b_{m,n}H_m\left(\frac{\xi-\mu}{\sigma}\right)\cdot \tag{6}$$

When we adopt (2b) for the case of ξ with mean and standard deviation equal μ and σ, respectively, we obtain the following general formula for spectral coefficients for $T(\omega_n,\xi)$:

$$b_{m,n}=\frac{1}{\gamma_m}\int_{-\infty}^{\infty}T(\omega_n,\sigma\xi+\mu)H_m(\xi)\frac{exp\left(\frac{-\xi^2}{2}\right)}{\sqrt{2\pi}}d\xi\ \cdot \tag{7}$$

Then we convert (1) into the following form:

$$T(\omega_n,\sigma\xi+\mu)=\sum_{k=0}^{\infty}a_{k,n}H_k\left(\frac{\sigma\xi}{\sigma_0}+\frac{\mu-\mu_0}{\sigma_0}\right), \tag{8}$$

After substitution of (8) into (7) we obtain:

$$b_{m,n}=\frac{1}{m!}\sum_{k=0}^{\infty}a_{k,n}\int_{-\infty}^{\infty}H_k(g\xi+h)H_m(\xi)\frac{exp\left(\frac{-\xi^2}{2}\right)}{\sqrt{2\pi}}d\xi\ \cdot \tag{9}$$

where:

$$g = \frac{\sigma}{\sigma_0}, \tag{10}$$

$$h = \frac{\mu - \mu_0}{\sigma_0}. \tag{11}$$

The formula (9) takes the form (13), after we use the identity (12) [6]:

$$H_k(x+y) = \sum_{j=0}^{k} \binom{k}{j} \cdot y^{k-j} \cdot H_j(x), \tag{12}$$

$$b_{m,n} = \frac{1}{m!} \sum_{k=0}^{\infty} a_{k,n} \sum_{j=0}^{k} h^{k-j} \binom{k}{j} \int_{-\infty}^{\infty} H_j(g\xi) H_m(\xi) \frac{\exp\left(\frac{-\xi^2}{2}\right)}{\sqrt{2\pi}} d\xi. \tag{13}$$

The infinite integral in (13) can be calculated analytically and after some transformations can be partially tabulated what significantly improves its calculation efficiency. The final form of our universal expansion coefficients is given by:

$$b_{m,n} = \frac{1}{m!} \sum_{k=0}^{\infty} a_{k,n} \frac{h^k}{\left(\sqrt{1-g^{-2}}\right)^m} \sum_{j=0}^{k} \left(\sqrt{\frac{g^2-1}{h^2}} \right)^j Q(j,k,m), \tag{14}$$

where Q(j,k,m) is the factor that do not depend on μ or σ therefore can be tabulated to increase the efficiency of our universal expansion coefficients. For (j - m) = 0, 2, 4, 6 ..., we have:

$$Q(j,k,m) = \frac{\frac{k!}{(k-j)!}}{\left(\frac{j-m}{2}\right)! \left(\sqrt{2}\right)^{j-m}}. \tag{15}$$

while for the rest values of (j - m) factor Q(j,k,m) is 0.

In practice the upper infinite limit in a summation occurring in (14) can be replaced with a natural number which can equal from a few to over twenty depending on domain limits of a mean of ξ which are considered in a simulator (e.g. $0 \le \theta_\mu \le \pi$, $0.2\text{m} \le R_\mu \le 0.3\text{m}$) and the probability distribution of ξ.

It should be noted that when more than one variable is assumed to be stochastic at one time [1], the analogous procedure as the one presented through (1) – (15) holds true.

III. Simulation Examples

In this section we compare the polynomial chaos approach with the standard Monte Carlo one in a calculation of a mean and a standard deviation of exemplary $T(\omega_n, \xi)$ for a frequency band 1–10 GHz with respect to the stochastic behavior of parameter ξ. As we pointed out in the previous sections we present numerical examples for the case of a diffraction (creeping) ray transfer function [7]. We assume a stochastic behavior of θ while a radius of a cylinder is assumed to be known constant equal 0.25m. In the first example θ has a Gaussian distribution with a mean and a standard deviation equal 0.2 and 0.02, respectively. In the second case we change the mean and the standard deviation of θ to 1 and 0.1, respectively.

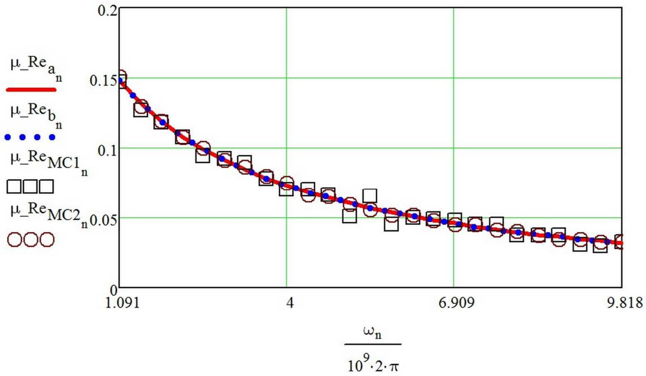

Fig. 2. Mean of a real part of an UTD creeping ray transfer function with respect to frequency when θ has a Gaussian distribution with μ=0.2 rad and σ=0.02 rad. MC results shown with squares and circles correspond to a number of samples 10 and 100, respectively.

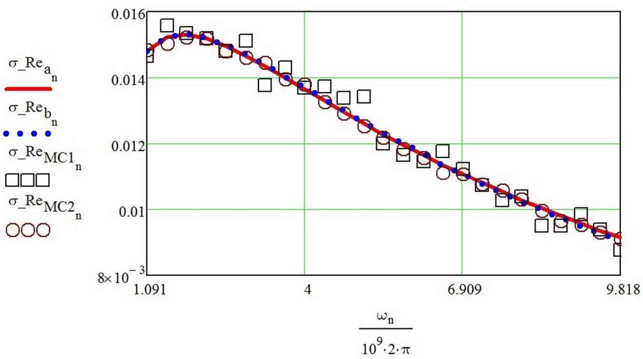

Fig. 3. Standard deviation of a real part of an UTD creeping ray transfer function with respect to frequency when θ has a Gaussian distribution with μ=0.2 rad and σ=0.02 rad. MC results shown with squares and circles correspond to a number of samples 1000 and 10000, respectively.

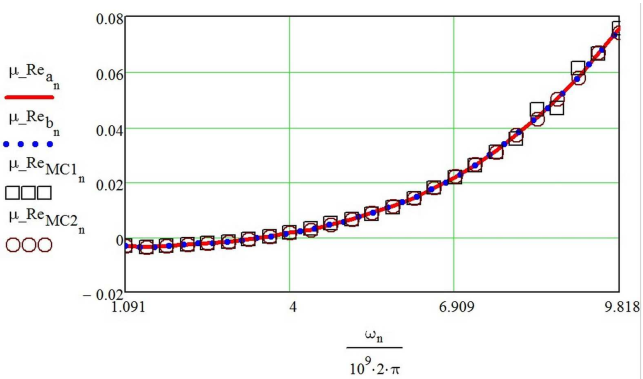

Fig. 4. Mean of a real part of an UTD creeping ray transfer function with respect to frequency when θ has a Gaussian distribution with μ=1 rad and σ=0.1 rad. MC results shown with squares and circles correspond to a number of samples 1000 and 10000, respectively.

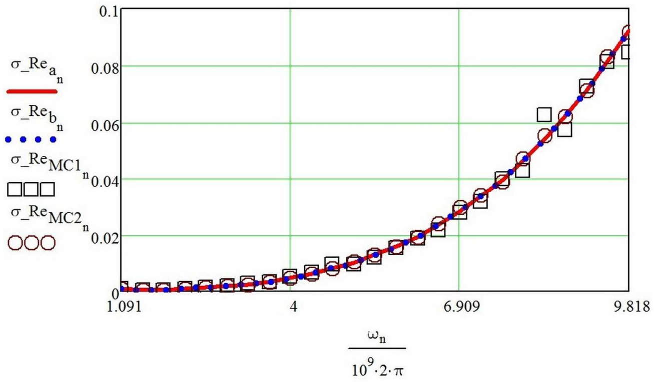

Fig. 5. Standard deviation of a real part of an UTD creeping ray transfer function with respect to frequency when θ has a Gaussian distribution with μ=1 rad and σ=0.1 rad. MC results shown with squares and circles correspond to a number of samples 1000 and 10000, respectively.

We present the results in Figs. 2-5. Each figure contains 4 graphs. The solid line corresponds to the results obtained by numerical calculation of (7). The dotted line relates to the results derived using (14). The square and circle symbol graphs correspond to Monte Carlo (MC) simulation results for different number of samples used. For space saving issues only real part of the functions is shown. We can see that in Figs. 2-5 the results obtained with our universal approach are in a very good agreement with the results obtained using numerical calculation of (7) (solid lines). This numerical calculation of (7) was necessary for each of the two examples. In our universal approach we do not need to apply time consuming numerical calculations. The time efficiency of our approach is over 30 times higher than for the case of a numerical calculation of (7) and over 300 times higher than for the Monte Carlo method regarding to e.g. standard deviation results from Fig. 3 and Fig. 5. Although we presented the simulation examples for the case of one convex obstacle the method can be easily adopted to the case of more obstacles in an UWB channel [5].

IV. Conclusions

In the paper we presented the new universal approach to a polynomial chaos expansion for simulation of EM wave propagation in an UWB channel that concentrates on statistical analysis of an EM field distribution. We presented and examined the new universal approach for the case of a diffraction phenomenon on a convex obstacle, which is a common element of an indoor propagation channel. The universality of our results express in a high accuracy of our expansion coefficients for a wide range of input stochastic variables without the need of performing time consuming numerical calculations. The coefficient are the functions of a mean and a standard deviation of a stochastic variable ξ. The coefficients (2) that are used to form the general coefficients (14) need to be calculated only once for each frequency sample of a considered UWB spectrum and then tabulated. Application of the vector fitting algorithm for an approximation of (2) in the frequency-domain allows to obtain a very simple form (sum of exponential functions) of an impulse response corresponding to a given ray [5] what allows an application of very effective convolution algorithms with a given UWB signal.

Our new approach enables to obtain an EM field stochastic distribution in a very short time comparing to an application of the Monte Carlo method while it allows flexible settings of parameters of stochastic variables in a wide range. The results obtained using our universal approach are very accurate what is presented for the case of exemplary stochastic variable distributions in Figs. 2-5.

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