Step response calculation of VLSI low loss interconnect using S-parameters

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Abstract—**This document describes step response calculation based on scattering parameters. To obtain the analytical formula of step response we approximate S-parameters and then introduce some simplification in step response formula. We present the results of simulation of VLSI interconnect, the RLC parameters are calculated using IE3D.**

I. Introduction

The scattering parameters can be powerful description of interconnect parameters e.g. [1] and [2]. They can be efficiently used for time-domain simulation, since in the time domain they have almost an exponential form, and they disappear in a short time. It is possible to approximate them in various ways. For example, numerous versions of the model order reduction (MOR) [3] methods (e.g. Krylov-subspace techniques) are used to approximate the original parameters of the two-port. Other efficient approach is the rational function approximation known as the vector fitting (VF) [3]. Both these approaches give very exact approximation however they are rather complicated. The less accurate but simpler and faster analytical methods, which can be used both in frequency and time domain are also needed in interconnect simulation and modeling. The presented approach seems to meet this requirements.

The modeling and simulation of interconnects are important part of VLSI system design. The interconnect delays can have even more impact on VLSI performance then the gate delay. Transmission lines are often used as a physical model of interconnects. This model is a compromise between the Maxwell-equation model of interconnects and the lumped elements circuit model. Hence, transient analysis and simulation of transmission lines with nonlinear loads is one of the most important issues in modern fast digital circuits. The transmission lines are considered as two-ports or multiports (in the case of coupled lines) in the frequency domain, and their parameters, etc. represented by nonrational functions of frequency are found.

The low loss interconnects are higher levels interconnections such as clock lines. The characteristic for them is that they have the global resistance less than lossless interconnect impedance R/Z_0 is less than 1. Then they cannot be modeled by RC transmission line model [4]. This work use this assumption to approximate the scattering parameters of the interconnect and then calculate the step response.

The work is organized as follows. In the second section there are introduced the most important information about scattering parameters, and introduced the solutions for reduced scattering parameters formulas. In the third section the interconnect voltage response is calculated. In the fourth section there are presented the simulation for various parameters and we conclude in the last section.

II. Scattering Parameters Calculation

A. General information

Instead of voltage–current, we can use the forward and backward current waves as the dependent variables to describe the transmission line [1]. Then the current waves are defined as follows:

$$\begin{aligned} i_-(y,\tau) &= \frac{1}{2}\left(Y_c(y)u(y,\tau) - i(y,\tau)\right) \\ i_+(y,\tau) &= \frac{1}{2}\left(Y_c(y)u(y,\tau) + i(y,\tau)\right) \end{aligned}, \qquad (1)$$

Where

i *and* u - current, voltage along the line

y - space coordinate normalized in relation to the line length l

τ - temporal coordinate normalized in relation to the line delay

$Y_c(y)$ - characteristic admittance of the line

Calculation of scattering parameters requires introducing normalized current waves in the following form:

$$\begin{aligned} i_-^n(y,\tau) &= i_-(y,\tau)\sqrt{Z_c(y)} \\ i_+^n(y,\tau) &= i_+(y,\tau)\sqrt{Z_c(y)} \end{aligned}, \qquad (2)$$

Which are related to incident and reflected power waves a_1, a_2, b_1, b_2.

$$\begin{aligned} i_-^n(0,\tau) &= b_1(\tau) \\ i_-^n(1,\tau) &= a_2(\tau) \\ i_+^n(0,\tau) &= a_1(\tau) \\ i_+^n(1,\tau) &= b_2(\tau) \end{aligned}, \qquad (3)$$

In scattering parameters calculation we take into account the load and input as characteristic impedances, as presented on the picture (Figure 1.):

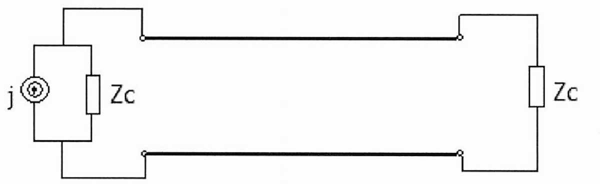

Fig. 1. The model of interconnect.

The exact form for the scattering parameters in Laplace domain can be expressed as follows:

$$S_1(p)=\frac{F_2(p)}{1+F_1(p)}\frac{1-e^{-2\Gamma(p)}}{1+\rho(p)e^{-2\Gamma(p)}},\qquad S_2(p)=\frac{1}{1+F_1(p)}\frac{2e^{-2\Gamma(p)}}{1+\rho(p)e^{-2\Gamma(p)}} \tag{4}$$

where:

$$F_1(p)=\frac{p+0.5\varepsilon}{\Gamma(p)},\quad F_2(p)=\frac{-0.5\varepsilon}{\Gamma(p)},$$

$$\Gamma(p)=\sqrt{(p+\varepsilon)p},\quad \rho(p)=\frac{\Gamma(p)-p-\varepsilon}{\Gamma(p)-0.5\varepsilon},\quad \varepsilon=\frac{R_t}{Z_c},$$

and R_t is total resistance of the line

The proposed solution allows to calculate the time domain scattering parameters. The scattering parameters in the Laplace domain are approximated by the method of successive approximations. Then we obtain the first approximation of the form:

$$S_{1\_1}(p)=\frac{\beta}{2}\frac{1}{p+\alpha}\left(e^{-2(p+\alpha)}-1\right),\qquad S_{2\_1}(p)=e^{-(p+\alpha)}, \tag{5}$$

where $\beta=\frac{R_w}{Z_c}$, $\alpha=\frac{C_t}{C_0}$, R_w is output resistance of input gate, C_0 is input capacitance of output gate, C_t is total capacitance of the line.

The comparison of approximated scattering parameters (5) and exact parameters (4) are presented in Fig 2-5

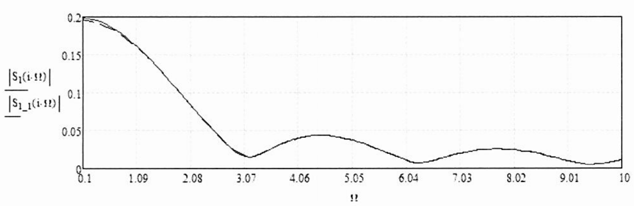

Fig. 2. Module of scattering parameter S_1 for single low loss interconnect (R=35Ω, L=2nH, C=12.4pF)

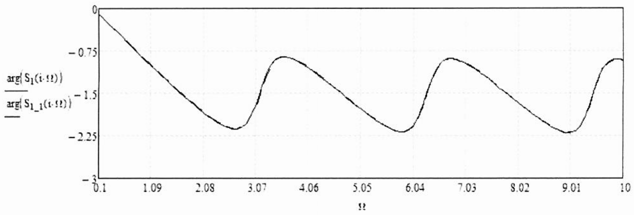

Fig. 3. Phase of scattering parameter S_1 for single low loss interconnect (R=35Ω, L=2nH, C=12.4pF)

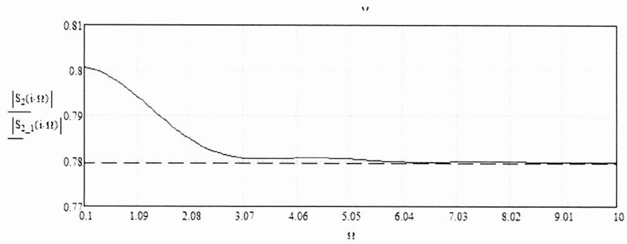

Fig. 4. Module of scattering parameter S_2 for single low loss interconnect (R=35Ω, L=2nH, C=12.4pF)

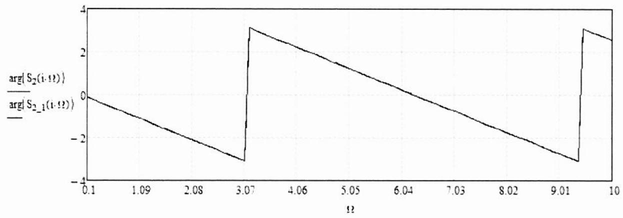

Fig. 5. Phase of scattering parameter S_2 for single low loss interconnect (R=35Ω, L=2nH, C=12.4pF)

We can see, that although the first approximation gives good results for both module and phase of S_1 and phase of S_2, the module of S_2 is not approximated properly. Then we calculate the second approximation of S_2 of the form:

$$S_{2\_2}(p)=e^{-(p+\alpha)}\cdot\left(1+\left(\frac{\alpha}{2(p+\alpha)}\right)^2\left(2(p+\alpha)-1\right)\right)+ \\ +\left(\frac{\alpha}{2(p+\alpha)}\right)^2\left(2(p+\alpha)-1\right)e^{-3(p+\alpha)}. \tag{6}$$

After second approximation is added the results are much better, but the form of the parameter is more complicated. In the next section we show that for step response calculation the improvement has an important impact for error decreasing.

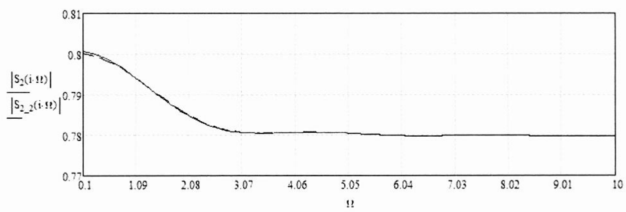

Fig. 6. Module of scattering parameter S_2 for single low loss interconnect for second approximation of scattering parameters (R=35Ω, L=2nH, C=12.4pF)

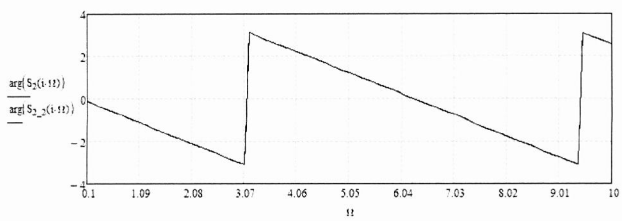

Fig. 7. Module of scattering parameter S_2 for single low loss interconnect for second approximation of scattering parameters (R=35Ω, L=2nH, C=12.4pF)

III. Step Response of Loaded Interconnect

The single interconnect driven by a gate and loaded by a gate can be modeled as transmission line with input resistance and output capacitance. The system of equation for RLC transmission line take the form:

$$\begin{cases} -\dfrac{\partial v}{\partial x} = Ri + L\dfrac{\partial i}{\partial t}, \\ -\dfrac{\partial i}{\partial x} = C\dfrac{\partial v}{\partial t}, \end{cases}$$

$$\begin{aligned} & i(x,0) = 0, \\ & v(x,0) = 0 \\ & e(t) - R_w i(0,t) = v(0,t), \\ & i(d,t) = \frac{\partial v(d,t)}{\partial t}. \end{aligned} \tag{6}$$

The step response of such model can be written as:

$$u(t) = E \cdot \int_0^t h(\tau)d\tau \tag{7}$$

and the h(t) we obtain from inverse Laplace transformation of H(p) written as:

$$H(p) = \frac{Z_c}{R_w + Z_c} \frac{(1+\rho_0)\cdot S_2}{S_2 \cdot \left(\dfrac{(\rho_w + \rho_0)}{(\rho_w \cdot \rho_0)} - S_1^2 + S_2^2\right)} \tag{8}$$

Where ρ_w and ρ_0 are input and output reflection coefficients respectively.

We can rewrite the formula to the form:

$$H(p) = \frac{2\alpha e^{-\frac{\varepsilon}{2}-p}}{(\beta+1)(\alpha+p)} \sum (q)^2 \tag{9}$$

Where:

$$q = \frac{\varepsilon\left(e^{-\varepsilon-2p}-1\right)(p-\beta\alpha)}{(\varepsilon+2p)(\beta+1)(\alpha+p)} - \frac{\varepsilon^2\left(e^{-\varepsilon-2p}-1\right)^2(\alpha-p)(\beta-1)}{4(\varepsilon+2p)^2(\beta+1)(\alpha+p)} + \frac{e^{-\varepsilon-2p}(\alpha-p)(\beta-1)}{(\beta+1)(\alpha+p)}$$

Then for first approximation of scattering parameters we obtain:

$$h_0(t) = \frac{2\alpha e^{-\frac{\varepsilon}{2}-\alpha(t-1)}}{(\beta+1)} \Phi(t-1) \tag{10}$$

Because of high complicated form of higher terms in (9) we decide to use only the steady state part of higher order approximation, then we obtain for the step response:

$$u_{\_1}(t) = E \cdot \left(Ih_0(t) + Ih_1(t) + Ih_{ss}(t)\right) \tag{11}$$

and

$$Ih_0(t) = \frac{2e^{-\frac{\varepsilon}{2}}}{(\beta+1)} \left(e^{-\alpha(t-1)} - 1\right)\Phi(t-1) \tag{12}$$

The second term in u(t) can be expressed as

$$Ih_1(t) = Ih_{11}(t) + Ih_{12}(t) + Ih_{13}(t) \tag{13}$$

$$Ih_{11}(t) = I_1(t-3)e^{-\frac{3\varepsilon}{2}} - I_1(t-1)e^{-\frac{\varepsilon}{2}} \tag{14}$$

$$\begin{aligned} I_1(t) = \Bigg[& \frac{8\beta\alpha^2 + 4\varepsilon\alpha}{(\varepsilon-2\alpha)^2(\beta+1)^2} e^{-\frac{\varepsilon t}{2}} + \\ & - e^{-\alpha t}\left(\frac{4\varepsilon\alpha - 2\beta\varepsilon^2 + 8\beta\alpha\varepsilon}{(\varepsilon-2\alpha)^2(\beta+1)^2}\right) + \\ & + \frac{t\left(2\alpha\varepsilon^2 - 4\varepsilon\alpha^2 + 2\beta\alpha\varepsilon^2 - 4\beta\alpha^2\varepsilon\right)}{(\varepsilon-2\alpha)^2(\beta+1)^2} + \\ & - \frac{\left(8\beta\alpha^2 - 8\beta\alpha\varepsilon + 2\beta\varepsilon^2\right)}{(\varepsilon-2\alpha)^2(\beta+1)^2} \Bigg]\Phi(t) \end{aligned} \tag{15}$$

$$Ih_{12}(t) = I_2(t-1)e^{-\frac{\varepsilon}{2}} - 2I_2(t-3)e^{-\frac{3\varepsilon}{2}} + I_2(t-5)e^{-\frac{5\varepsilon}{2}} \tag{16}$$

$$\begin{aligned} I_2(t) = & \frac{2\alpha\varepsilon^2(\beta-1)}{4(\beta+1)^2\alpha\varepsilon^2(\varepsilon-2\alpha)^3} \cdot \\ & \cdot \Big\{\left[-\left(12\alpha^2\varepsilon - 8\alpha^3 + 4\alpha\varepsilon^2\right) - t\cdot\left(\alpha\varepsilon^3 - 4\alpha^3\varepsilon\right)\right]e^{-\frac{\varepsilon t}{2}} + \\ & + 12\alpha^2\varepsilon - 8\alpha^3 - 6\alpha\varepsilon^2 + \varepsilon^2 \\ & - \left(\varepsilon^3 - 10\alpha\varepsilon^2 + t\cdot\left(2\alpha\varepsilon^3 - 4\alpha^2\varepsilon^2\right)\right)e^{-\alpha t}\Big\}\Phi(t) \end{aligned} \tag{17}$$

$$Ih_{13}(t) = I_3(t-3) \tag{18}$$

$$I_3(t) = \frac{2e^{-3\frac{\varepsilon}{2}}(\beta-1)}{(\beta+1)^2} \cdot \left[\left(1 - 6\alpha + 2\alpha t\right)e^{-\alpha t} - 1\right]\Phi(t) \tag{19}$$

the steady state part of higher order approximation, take the form:

$$Ih_{ss}(t) = \sum_{n=2}^{N} \left\{ \frac{2e^{-\frac{\varepsilon}{2}}}{\beta+1} \left[\frac{3\beta + 1 + (2\beta-6)e^{-\varepsilon}}{4(\beta+1)} + \frac{(1-\beta)e^{-2\varepsilon}}{4(\beta+1)} \right]^n \right\} \Phi(t-2n-1) \tag{20}$$

The simulation results are presented in next section.

As we remember from Section II The first approximation of scattering parameter S_2 is not sufficient, then we try to calculate the output response using the second approximation of S_2. We will not present particular formulas here, because they are quite long. The voltage response:

$$u_{\_2}(t) = E \cdot \left(Ih_{0\_2}(t) + Ih_{1\_2}(t) + Ih_{ss\_2}(t)\right) \tag{21}$$

We present simulation results in next section.

IV. Simulation Results

Figures 9 - 11 present the voltage response calculated with first scattering parameter approximation (dotted curve) and with first S_1 and second S_2 parameter approximation(11) compared with exact voltage response (solid) and with first

S_1 and second S_2 approximation (dashed curve) taken from (21). We can see, that the more low loss the interconnect the more accurate is when we take only first approximation.

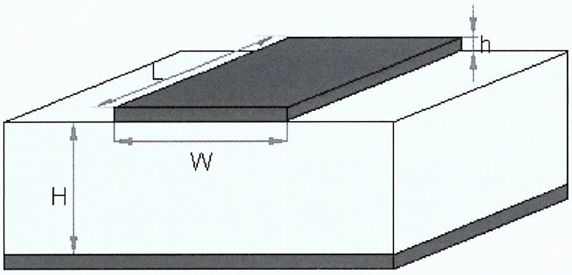

Fig. 8. Geometrical model used in the example. W= 2μm, H=300μm, h=1μm, l=5mm, ε_{Si}=11.9, σ_{Si}=10000S/m, σ_{Cu}= 2.73e+7S/m

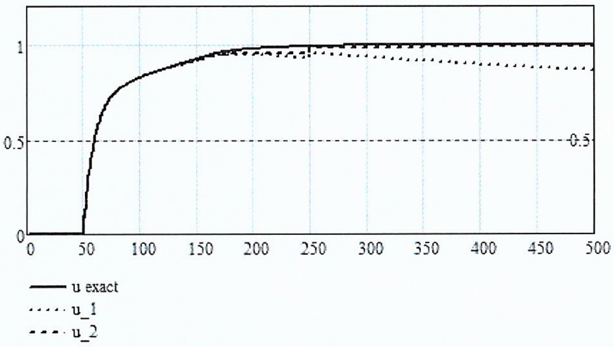

Fig. 9. Step voltage response for single low loss interconnect (R=124Ω, L=0.2nH, C=pF)

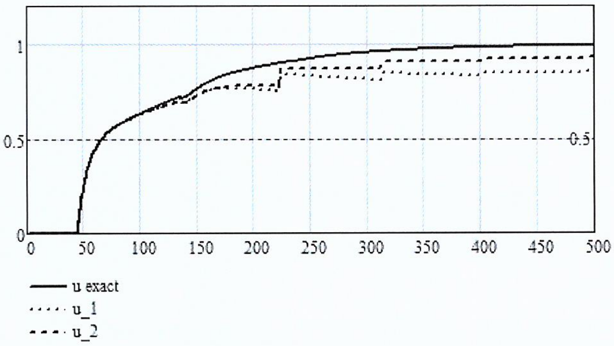

Fig. 10. Step voltage response for single low loss interconnect (R=35Ω, L=2nH, C=12.4pF)

The example parameters are taken from literature (fig10-11) and calculated with IE3D simulator (in the Fig 9). The IE3D simulations are done for the structure presented in Fig.8 The simulation result R_t=124Ω, C_t=0.2pF, L_t=12.4nH, than the ε=0.494. We assume the modeled inverter output resistance R_w=25Ω, and input capacitance C_0=0.1pF.

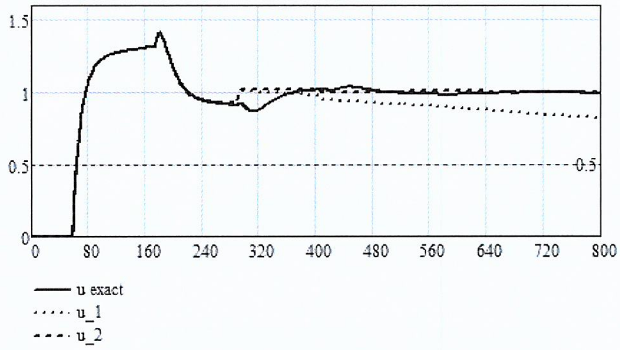

Fig. 11. Step voltage response for single low loss interconnect (R=10.5Ω, L=2.9nH, C=1.17pF)

V. Conclusions

In the paper we present the analytical method of step response for VLSI interconnect calculation. The higher level interconnects characterize the high inductance compared to resistance, than we can use several approximation to obtain the analytical formula. As we present the results are good enough to $t_{50\%}$ calculation or signal shape observation. The analytical formula enables very fast and efficient calculations.

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