Sensitivity of output response to geometrical dimensions in VLSI interconnects

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Abstract

The paper presents a sensitivity analysis of step and ramp response of VLSI interconnects to geometrical dimensions of microstrip model of higher level interconnects. The method is based on analytical form of VLSI output response calculated using multiple scales method.

Introduction

Intensive development of VLSI systems means faster circuits and smaller die size. Special problem in miniaturization is considered with signal propagation between elements. The minimalization of crosstalks, delays and signal deformation in interconnects are still a very important task to solve in VLSI design [1,2]. In modern integrated circuits there are many layers of interconnects with different geometrical structures, which allows signal propagation on specific distance. Optimal design needs applicable methods of simulation. In recent years, there is of particular interest in the methods of calculation the time delay for a single interconnect or a system of two or three coupled interconnects.

A particular case are higher level interconnects, in which, due to the large length, the technology needs to reduce the loss, e.g. increasing cross-section, resulting in a decrease of resistance of the interconnect. The analysis of such system can be simplified to the analysis of low-loss interconnects. In this case, we can assume:

$$\frac{R_t}{Z_0} < 1 \tag{1}$$

Where R_t –total resistance of interconnect, Z_0 – characteristic impedance of lossless line, modelling this interconnect.

Higher level interconnects are also characterized by greater inductance. The digital signal representing one bit can be considered as a superposition of two step signals. The spectrum of this signal is bandlimitted by $1/T_r$, and T_r is rise time of signal, in an ideal case equal to zero. In practice, the time T_r is different from zero, so frequently there are used algorithms, using ramp response as output response. Interconnect inductance affect significantly the shape of the output response and in the signal at the end of interconnect occurs overshoot for the first travelling wave if resistance of the input gate is smaller than the total resistance of the interconnect. It allows to simplify analysis of such interconnects to analysis of first wave only. Typically for higher level interconnects simulation there is RLC transmission line model taken with microstrip geometrical model. The criteria for determining the need for such a model can be found e.g. in [3].

In the paper we present the method of calculating the step and ramp response directly from transmission line differential equation constructed for of single interconnect loaded by input gate capacitance and powered by gate with output resistance. Such system was considered e.g. in [2,4]. Methods presented in that works are generally based on some set of earlier measurement or simulation results and are partially obtained heuristically. There is not always possible to calculate the sensitivity of step or ramp responses to interconnect parameters in analytical way. In our method it is possible to derive in a good accuracy the step and ramp responses analytical formulas. Hence there is a possibility to find sensitivity to RLC parameters directly from sensitivity definition. RLC parameters can be expressed by known [5] analytical formulas containing geometrical dimensions and physical parameters of the whole structure, so there is possibility to calculate the sensitivity of output response to geometrical dimensions too.

In the section 2 we present the method of calculating of closed form formula for step and ramp response of interconnect using multiple scales method. In section 3 the sensitivity calculations are presented. In section 4 we present the simulation results. Conclusions are presented in the last section.

Output response calculation

In our work we derive the method for compute output response for higher level interconnect assuming RLC transmission model (Fig.1.) and the global resistance less than lossless interconnect impedance. That assumption allows treat the line as low loss line with high inductance influence.

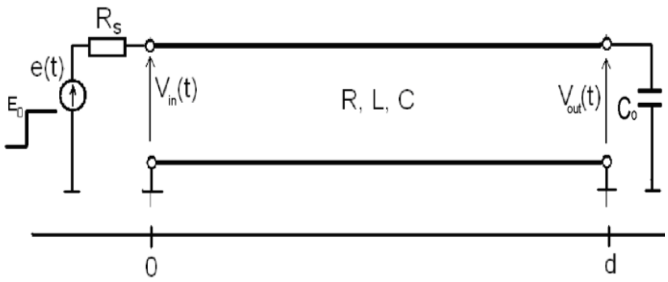

Fig. 1 The model of the considered system

For the system of differential equation (2) with initial and boundary conditions (3,4) we could use the perturbation method of multiple scales, using the $\varepsilon = \frac{R_t}{Z_0}$ as perturbation parameter ($R_t = R \cdot d$).

$$\begin{cases} -\frac{\partial v}{\partial x} = Ri + L\frac{\partial i}{\partial t}, \\ -\frac{\partial i}{\partial x} = C\frac{\partial v}{\partial t} \end{cases} \tag{2}$$

$$i(x,0)=0, \quad v(x,0)=0, \tag{3}$$

$$e(t)-R_s i(0,t)=v(0,t),$$

$$-i(d,t)=C_0\frac{\partial v(d,t)}{\partial t}, \tag{4}$$

where

R, L, C – line parameters, C_0 – input inverter capacitance, R_s – output inverter resistance, i(x,t), v(x,t) – current and voltage in line, respectively, d – line length, t, x – time and space variable, respectively.

To simplify the calculation and enable the perturbation parameter occur in differential equation we are scaling variables occurring in system (1):

$$y=\frac{x}{d},\ \tau=\frac{t}{\sqrt{L_t C_t}},\ \tilde{v}=-\sqrt{\frac{C_t}{L_t}}v,\ \tilde{e}=\sqrt{\frac{C_t}{L_t}}e,\ \beta=\sqrt{\frac{C_t}{L_t}}R_s$$

and we can rewrite (2) and (3) in the following way

$$\frac{\partial\tilde{v}}{\partial y}=\varepsilon i+\frac{\partial i}{\partial\tau}, \qquad \frac{\partial i}{\partial y}=\frac{\partial\tilde{v}}{\partial\tau}, \tag{5}$$

$$\tilde{e}(\tau)-\beta i(0,\tau)=-\tilde{v}(0,\tau),$$

$$-i(1,\tau)=\frac{C_0}{C_t}\frac{\partial\tilde{v}(1,\tau)}{\partial\tau}. \tag{6}$$

The way of calculating the step output response for such a system is presented in [3]. For the first travelling wave we obtain:

$$v_{out}(d,t)=v_{01}(d,t)+v_{11}(d,t)=$$

$$=\frac{E_0}{\beta+1}\left(A\cdot\left(\frac{t-T}{T}\right)\cdot e^{-\frac{\alpha}{T}(t-T)}-B\cdot\left(1-e^{-\frac{\alpha}{T}(t-T)}\right)\right)\cdot\mathbf{1}(t-T), \tag{7}$$

for time 0<t<3T

where:

$$A=-\varepsilon\cdot\alpha\cdot e^{-1.5\varepsilon},\ B=\frac{\varepsilon}{2}e^{-0.5\varepsilon}\left(1+e^{-\varepsilon}\right)-\frac{\beta\varepsilon}{\beta+1}-2e^{-0.5\varepsilon}$$

and the ramp output response:

$$v_{out}^{r}(t)=\frac{E_0}{(\beta+1)T_r}\cdot\left[k\left(1-e^{-\alpha\frac{t-T}{T}}\right)-\frac{t-T}{T}\left(B+\frac{A}{\alpha}e^{-\alpha\frac{t-T}{T}}\right)\right],$$

$$for \quad t-T\le T_r$$

$$v_{out}^{r}(t)=\frac{E_0}{(\beta+1)T_r}\cdot\left[\begin{array}{l}\left(\left(\frac{t-T-T_r}{T\alpha}\right)A+k\right)e^{-\alpha\left(\frac{t-T_r-T}{T}\right)}\\ -\left(\frac{t-T}{T\alpha}A+k\right)e^{-\alpha\frac{t-T}{T}}-\frac{T_r}{T}B\end{array}\right], \tag{8}$$

$$for \quad t-T\ge T_r$$

where $k=\left(\frac{A}{\alpha^2}+\frac{B}{\alpha}\right)$.

Sensitivity analysis

The interconnect response both for step and ramp responses can be highly sensitive to small changes in geometrical dimensions. Due to generation by the presented approach the closed form formulas for output response (7) and (8), we could calculate the sensitivity of the output response v to parameter λ with the formula:

$$S_v^{\lambda}=\frac{\lambda}{v}\frac{\partial v}{\partial\lambda} \tag{9}$$

The sensitivity (9) was compared with the sensitivity of the PSpice simulations for the RLC parameter analysis. The sensitivity in PSpice program we compute using the output signal with perturbed parameter to which the sensitivity is calculated. We use the following formula to calculate the sensitivity of the ramp response in PSpice:

$$S_{vSPICE}^{\lambda}=\frac{v(t,\lambda)-v(t,\lambda\cdot1.01)}{v(t,\lambda)}100 \tag{10}$$

The sensitivity for the step response obtained from (9) compared with (10) gives very good agreement for RLC parameters. The exemplary result for the sensitivity of ramp response to resistance of the interconnect is presented in Fig. 2.

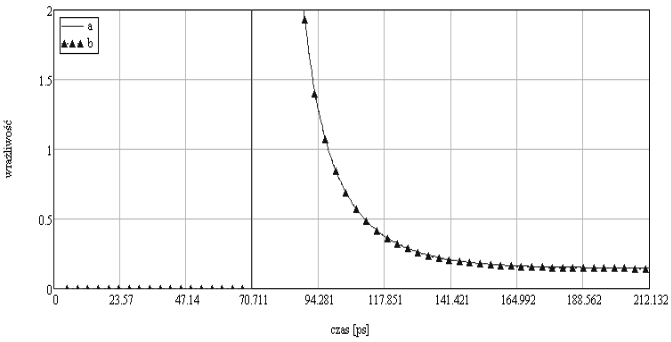

Fig. 2. Sensitivity of the ramp response to the total capacitance of the interconnect (R_t=25Ω, L_t=5nH, C_t=1pF, C_0=0.5pF and R_s=25Ω, it means ε=0.354, α=2, β=0.354) a – sensitivity from (10), b – sensitivity from (9)

In the case of microstrip interconnect model presented in Fig. 2 it is possible to find the relationship between the RLC and geometrical dimensions W, h, L, and H.

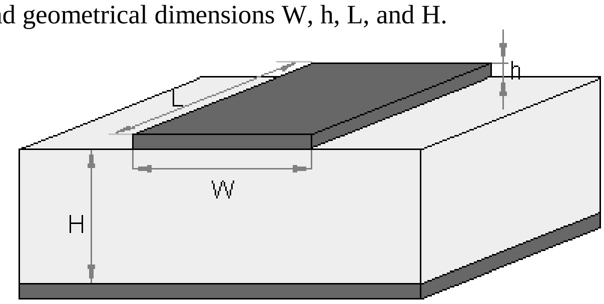

Fig.3. Microstrip geometrical model.

Using the formulas for R, L, C parameters based on geometrical dimensions taken from [5] we can present the sensitivity of step response of the interconnect to the parameter represented width of the interconnect as:

$$S^W_{v_s} = \frac{W}{v_s}\frac{\partial v_s}{\partial W} = \\ = \frac{W}{v_s}\frac{\partial}{\partial W}\left[\frac{E_0}{\beta+1}\left(A\tilde{t}\,e^{-\alpha\tilde{t}} - B\cdot\left(1-e^{-\alpha\tilde{t}}\right)+C\tilde{t}\right)\right] \qquad (11)$$

where $\alpha, \beta, A, B, C, \tilde{t}$ are dependent on RLC, and then on geometrical dimensions and L, C parameters are given by the formulas [5]:

$$C = \varepsilon\left(\frac{W-h/2}{H} + \frac{2\pi}{\ln\left(1+\frac{2H}{h}\left(\frac{2H}{h}+2\right)\right)}\right)$$

$$for\; W > h/2 \qquad (12)$$

$$C = \frac{W}{H} + \frac{\pi\left(1-0.0543\frac{h}{2H}\right)}{\ln\left(1+\frac{2H}{h}+\sqrt{\frac{2H}{h}\left(\frac{2H}{h}+2\right)}\right)} + 1.47$$

$$for\; W < h/2$$

$$L = 5.0832\cdot 10^{-9}\, d\,\ln\left[\frac{8H}{W+0.4h\left(\ln\left(\frac{12.57W}{h}\right)+1\right)} + \frac{W+0.4h\left(\ln\left(\frac{12.57W}{h}\right)+1\right)}{4H}\right] \qquad (13)$$

We can calculate the derivative of step response as:

$$\frac{dv_s}{dW} = \frac{\partial v_s}{\partial R}\frac{\partial R}{\partial W} + \frac{\partial v_s}{\partial L}\frac{\partial L}{\partial W} + \frac{\partial v_s}{\partial C}\frac{\partial C}{\partial W} \qquad (14)$$

Then the sensitivity formula can be calculated from:

$$S^W_{v_s} = \frac{W}{v_s}\frac{dv_s}{dW} = \frac{W}{v_s}\left(\frac{\partial v_s}{\partial R}\frac{\partial R}{\partial W} + \frac{\partial v_s}{\partial L}\frac{\partial L}{\partial W} + \frac{\partial v_s}{\partial C}\frac{\partial C}{\partial W}\right) \qquad (15)$$

In similar way we can derive the sensitivity of the step response to dimension H:

$$S^H_{v_s} = \frac{H}{v_s}\frac{dv_s}{dH} = \frac{H}{v_s}\left(\frac{\partial v_s}{\partial R}\frac{\partial R}{\partial H} + \frac{\partial v_s}{\partial L}\frac{\partial L}{\partial H} + \frac{\partial v_s}{\partial C}\frac{\partial C}{\partial H}\right) \qquad (16)$$

Similar consideration can be done for the ramp response. The sensitivity of the ramp response to width of the line W can be calculated from:

$$S^W_{v_r} = \frac{W}{v_s}\frac{dv_r}{dW} = \frac{W}{v_r}\left(\frac{\partial v_r}{\partial R}\frac{\partial R}{\partial W} + \frac{\partial v_r}{\partial L}\frac{\partial L}{\partial W} + \frac{\partial v_r}{\partial C}\frac{\partial C}{\partial W}\right) \qquad (17)$$

The sensitivity of the ramp response to the distance between the ground and the wire H is derived as:

$$S^H_{v_r} = \frac{H}{v_s}\frac{dv_r}{dH} = \frac{H}{v_r}\left(\frac{\partial v_r}{\partial R}\frac{\partial R}{\partial H} + \frac{\partial v_r}{\partial L}\frac{\partial L}{\partial H} + \frac{\partial v_r}{\partial C}\frac{\partial C}{\partial H}\right) \qquad (18)$$

Derivation of $\frac{\partial v_s}{\partial R}, \frac{\partial v_s}{\partial L}, \frac{\partial v_s}{\partial C}, .. \frac{\partial R}{\partial W}, \frac{\partial L}{\partial W}, \frac{\partial L}{\partial H}$ etc. are simple but obtained formulas in most cases are very large, as an example we adduce:

$$\frac{\partial v_s}{\partial R} = \frac{E_0}{\beta+1}\frac{1}{Z_0}\cdot\left[\begin{array}{l}(1+0.5\varepsilon)e^{-0.5\varepsilon}\tilde{t}\,e^{-\alpha\tilde{t}} + \frac{\beta}{\beta+1}\tilde{t} + \\ -\frac{1}{\alpha}\left((1+\alpha-0.5\varepsilon)e^{-0.5\varepsilon} - \frac{\beta}{\beta+1}\right)\left(1-e^{-\alpha\tilde{t}}\right)\end{array}\right]$$

$$\frac{\partial R}{\partial W} = -\frac{\rho d}{W^2 h}$$

$$\frac{\partial L}{\partial W} = \frac{5.0832\cdot 10^{-9}\, d\left(\frac{0.4h}{W}+1\right)\cdot X^2 - 32h^2}{W+0.4h\left(\ln\left(\frac{12.57W}{h}\right)+1\right)X^2 - 32h^2},$$

where $X = \left(W+0.4h\left(\ln\left(\frac{12.57W}{h}\right)+1\right)\right)$

The simulation results are presented in the next section.

Simulation results

Below (Fig.4-7) we present some simulation results for typical microstrip model parameters. At Fig.4-5 we present the sensitivity of step response and ramp response respectively to width of interconnect. For both responses the simulations show the similar results, however the ramp response is a little more sensitive. The most sensitive is the response in the first moment after it arrives to the end of the interconnect. Later the sensitivity is significantly smaller. The reasons are inductance and capacitance changes during the geometrical dimension changes and then the delay time also change.

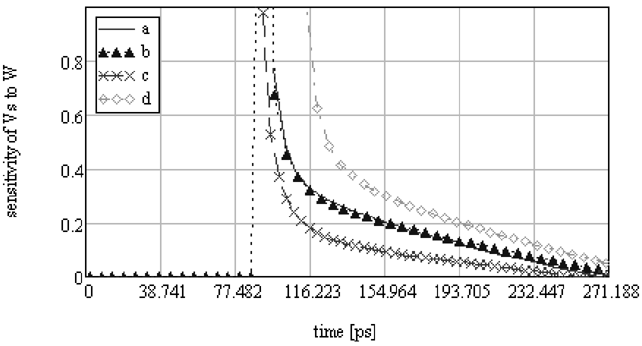

Fig. 4. Comparison of sensitivity of step response to width of the interconnect for exemplary parameters a - C_0=0.5pF and R_s=25Ω, W=2μm, h=1μm, H=300μm, d=2mm, b- C_0=0.5pF and R_s=25Ω, W=2μm, h=1μm, H=600μm, d=2mm, c- C_0=0.5pF and R_s=25Ω, W=4μm, h=1μm, H=300μm, d=2mm, d- C_0=0.5pF and R_s=25Ω, W=2μm, h=1μm, H=300μm, d=2.4mm.

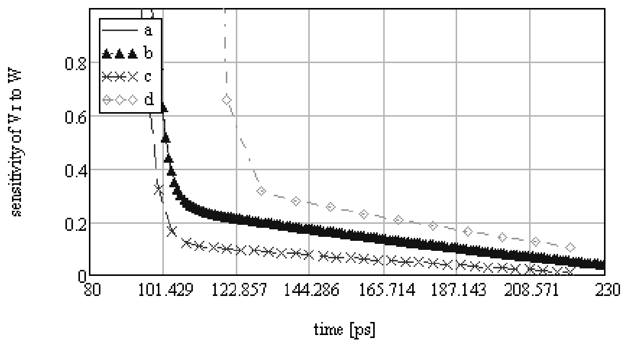

Fig. 5. Comparison of sensitivity of ramp response to width of the interconnect for exemplary parameters a - C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=300µm, d=2mm, b- C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=600µm, d=2mm, c- C_0=0.5pF and R_s=25Ω, W=4µm, h=1µm, H=300µm, d=2mm, d- C_0=0.5pF and R_S=25Ω, W=2µm, h=1µm, H=300µm, d=2.4mm.

The sensitivity of output response to the distance between wire and ground (Fig. 6-7) is quite small ($S_v^H < 2\%$) and it does not change a lot during interconnect dimensions change.

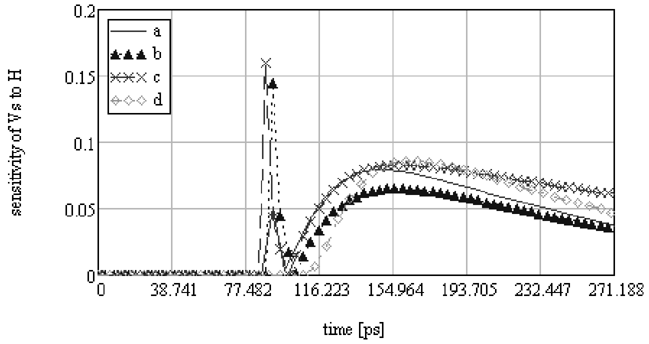

Fig. 6 Comparison of sensitivity of step response to H for exemplary parameters a - C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=300µm, d=2mm, b- C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=600µm, d=2mm, c- C_0=0.5pF and R_s=25Ω, W=4µm, h=1µm, H=300µm, d=2mm, d- C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=300µm, d=2.4mm.

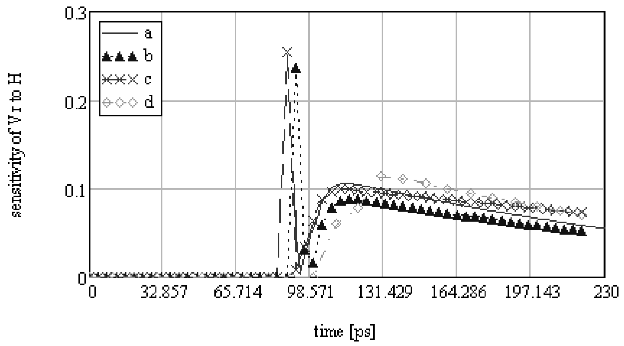

Fig. 7. Comparison of sensitivity of ramp response to H value for exemplary parameters a - C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=300µm, d=2mm, b- C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=600µm, d=2mm, c- C_0=0.5pF and R_s=25Ω, W=4µm, h=1µm, H=300µm, d=2mm, d- C_0=0.5pF and R_s=25Ω, W=2µm, h=1µm, H=300µm, d=2.4mm.

Conclusions

In the paper the sensitivity analysis of the step and ramp response to geometrical dimensions of the interconnect are presented. We apply the interconnect sensitivity calculation to microstrip higher level interconnect model. The sensitivity is calculated directly from sensitivity definition, due to analytical closed form formula of the step and ramp responses for the first traveling wave obtained with multiple scales method. Then it is possible to obtain formulas for sensitivity analysis. The method is accurate for higher level low-loss, highly inductive interconnects.

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