# SENSITIVITY ANALYSIS OF THE OUTPUT SIGNAL OF VLSI INVERTER-INTERCONNECT-INVERTER SYSTEM TO SELECTED PARAMETERS

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**ABSTRACT:** Interconnect parameters play an important role in signal propagation in VLSI systems. In the paper we present the sensitivity analysis to parameters of typical on-chip interconnects for ramp response. In the paper we show also the sensitivity to parameters for the threshold crossing time. We compute the output response of the interconnect and the threshold crossing time formulas based on the multiple scales method.

## INTRODUCTION

The term sensitivity characterizes the influence of changing parameter value on system functionality. Sensitivity analysis permits to estimate, which of system parameters affect more and which one of them less on it. From the designer of VLSI circuits point of view this permits for identification of the number of variables, which have critical influence on operation of the circuit. For sensitivity analysis calculations numerical methods can be used. Changing value of various variables in the circuit one determines importance and influence of each variable of the circuit [1]. The problem of sensitivity analysis of single or coupled interconnects has been considered in paper [2]. The authors have proposed effective method of optimization permitting for selection of the trace width in a way, which minimize the delay of the signal. The method is based on calculation of sensitivity of interconnect delay using the method of moments. In [3] authors use the sensitivity of the system for optimization of parameters. Sensitivity analysis is carry out in frequency domain. One of the basic works concerning sensitivity analysis is [4]. The authors introduce equations describing sensitivity from transmission line equations and build up the model for sensitivity simulation in time domain.

Faster circuits, higher frequencies, smaller die sizes in VLSI modern technology causes new problems in system designs. During that process there is a need to appropriate model assumption to ensure that the simulation of the system will give good accuracy. Both the gates and interconnect must be considered during system modelling. Due to the resistance of the modern interconnects are smaller than the lossless line impedance $Z_0$ (1) there is no longer possible to model the interconnect by RC transmission line, and the RLC transmission line must be considered [5,6]

$$\frac{R}{2Z_0} \leq 1.$$

(1)

Although the high growth of device speeds the on-chip interconnect has not been scaling so fast. Especially global and clock wires could be 2-8x the minimum dimensions [5]. The higher level

clock interconnects might have total resistance 8.2-21.5$\Omega$ for 1 mm line length. Most of the application for simulation use only RC model, and that approach does not give good accuracy [5]. Typical simulation problems consider simulation of the one interconnect or more coupled interconnects.

In previous our work [7] we derive the method for calculation the step and ramp responses for clock and global wires assuming RLC transmission model and the global resistance less than lossless interconnect impedance. That assumption allows treat interconnect as low-loss transmission line with high inductance influence. For the system of transmission line differential equations with initial and boundary conditions we could use the perturbation method of multiple scales, using the $\varepsilon = \dfrac{R_t}{Z_0}$ as perturbation parameter ( $R_t = R \cdot d$ ).

In our work we present sensitivity analysis based on the previously derived, by the method of multiple scales, relationships for the voltage at output of interconnect in the system inverter – interconnect – inverter. Simulations used in our investigation of the interconnect response, both the output response and threshold crossing time calculation can be highly sensitive to small changes in the parameter values.

The paper is organized as follows: in the second section there we shortly present the method of step and ramp response calculation and we show how to obtain the threshold crossing time from the output response. In the next section we present the sensitivity analysis of the output response and threshold crossing time for typical on-chips interconnects to interconnect parameters. We conclude in the last section.

## MODEL OF INVERTER-INTERCONNECT-INVERTER SYSTEM

In this section we shortly present the method of computation the step and ramp responses for clock and global wires assuming RLC transmission model  (Fig. 1) and the global resistance less than lossless interconnect impedance. That assumption allows treat the line as low-loss line with high inductance influence.
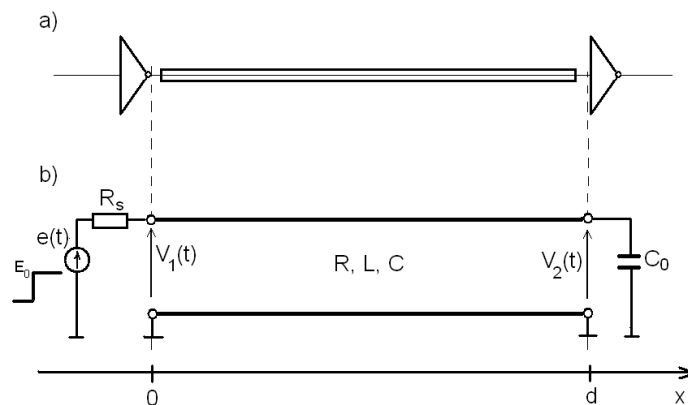


Fig.  1 The system inverter-interconnect-inverter.

For the system of differential equation (2) with initial and boundary conditions (3,4) we could use the perturbation method of multiple scales, using the  $\varepsilon = \dfrac{R_t}{Z_0}$ as perturbation parameter ( $R_t = R \cdot d$ ).

$$\begin{cases} -\dfrac{\partial v}{\partial x} = Ri + L\dfrac{\partial i}{\partial t}, \\ -\dfrac{\partial i}{\partial x} = C\dfrac{\partial v}{\partial t} \end{cases}$$

2)

$$i(x,0) = 0, \quad v(x,0) = 0,$$

3)

$$e(t) - R_s i(0,t) = v(0,t),$$

$$-i(d,t) = C_0 \frac{\partial v(d,t)}{\partial t},$$

4)

where

R, L, C – line parameters, $C_0$ – input inverter capacitance, $R_s$ – output inverter resistance, i(x,t), v(x,t) – current and voltage in line, respectively, d – line length, t, x – time and space variable, respectively.

After scaling system (2) to obtain the perturbation parameter in the system, we can observe, that resistance of the interconnect influence only for perturbation parameter value. After scaling we have:

$$y = \frac{x}{d}, \quad \tau = \frac{t}{\sqrt{L_t C_t}}, \quad \tilde{v} = -\sqrt{\frac{C_t}{L_t}}v,$$

$$\tilde{e} = \sqrt{\frac{C_t}{L_t}}e, \quad \beta = \sqrt{\frac{C_t}{L_t}}R_s$$

(5)

**Output response calculation**

The system (2-4) after scaling (5) can be rewritten as:

$$\frac{\partial \tilde{v}}{\partial y} = \varepsilon i + \frac{\partial i}{\partial \tau},$$

$$\frac{\partial i}{\partial y} = \frac{\partial \tilde{v}}{\partial \tau}$$

6)

$$\tilde{e}(\tau) - \beta i(0,\tau) = -\tilde{v}(0,\tau),$$

$$-i(1,\tau) = \frac{C_0}{C_t}\frac{\partial \tilde{v}(1,\tau)}{\partial \tau}$$

7)

.

The multiple scales method [6] requires expansion of the solution for differential equations into a power series of perturbation parameter, which values are relatively small. Additionally, new space variables are introduced. In our analysis we assume the line resistance in low resistance interconnects is small compared to the lossless line impedance Z0 (Z0>Rt), the perturbation parameter is ε, and we limit the expansion to two terms and two space variables. For the system of equations (6) we have

$$\tilde{v}(y,\tau) = \tilde{v}_0(y_0, y_1, \tau) + \varepsilon \cdot \tilde{v}_1(y_0, y_1, \tau),$$

$$i(y,\tau) = i_0(y_0, y_1, \tau) + \varepsilon \cdot i_1(y_0, y_1, \tau),$$

$$y_0 = y, \quad y_1 = \varepsilon \cdot y.$$

Than we obtain new systems of differential equation, the result of alternation equation depend on previous equation. The method allows to compute the lossless line output step response comply the losses parameter $\varepsilon$. The way of calculating the step output response for such a system is presented in [7]. For the first traveling wave we obtain:

$$v_s(d,t) = v_{01}(d,t) + \varepsilon v_{11}(d,t) =$$

$$= \frac{E_0}{\beta+1}\left( A_2 \frac{t-T}{T} e^{-\frac{\alpha}{T}(t-T)} - B_2 \cdot \left(1 - e^{-\frac{\alpha}{T}(t-T)}\right) + C_2 \frac{t-T}{T}\right) \cdot \mathbf{1}(t-T), \qquad 8)$$

for time 0<t<3T

where:

$$A_2 = \varepsilon \cdot e^{-0.5\varepsilon}, \quad C_2 = \varepsilon \frac{\beta}{\beta+1}, \quad B_2 = \left(\frac{\varepsilon}{\alpha} - 2\right)e^{-0.5\varepsilon} + \frac{\beta}{\beta+1}\frac{\varepsilon}{\alpha}.$$

The simulations show that the approximation gives good results, especially during the rise time of output signal. When the aim is to calculate the threshold crossing time or to predict the behavior of the interconnect in that range of time, the accuracy of formula (8) is enough.

The fast circuits generate the rising time of the input signal comparable to the time delay, therefore in many on-chip interconnects we have to take into account the influence of the input voltage rise time $T_r$. We model such a problem with the ramp excitation:

$$v_{in}(t) = \frac{E_0}{T_r} t \cdot 1(t) - \frac{E_0}{T_r}\left(t - T_r\right) \cdot 1(t - T_r) \qquad 9)$$

We must consider two cases – one for $0<t<T_r$, and the other for $t>T_r$. Denoting the response to ramp excitation at the end of the interconnect as $v_{out}^r$ and we have the final output voltage signal [7]:

$$v_r(t) = \frac{E_0}{(\beta+1)T_r}\left[ \frac{C_2}{2}\left(\frac{t-T}{T}\right)^2 - B_2 \frac{t-T}{T} + \left(K_1 \frac{t-T}{T} - K_2\right)e^{-\alpha\frac{t-T}{T}} + K_2 \right], dla \quad t-T \le T_i$$

$$v_r(t) = \frac{E_0}{(\beta+1)T_r}\left[ \begin{array}{l} K_1 \frac{t-T}{T} e^{-\alpha\frac{t-T}{T}} T_2 + C_2 \cdot T_r \frac{t-T}{T} - \left(K_2 \cdot T_2 - K_1 \cdot T_r e^{\alpha T_r}\right)e^{-\alpha\frac{t-T}{T}} \\ -\frac{C_2 \cdot T_r^2}{2} - B \cdot T_r \end{array} \right], \qquad 10)$$

$dla \quad t-T \ge T_r,$

$$K_1 = -\frac{A_2}{\alpha}, \quad K_2 = \left(\frac{B_2 - K_1}{\alpha}\right), \quad T_2 = 1 - e^{\alpha T_r}.$$

The presented approach allows calculating the approximated ramp excitation response. The exemplary ramp response is presented on Fig. 2.
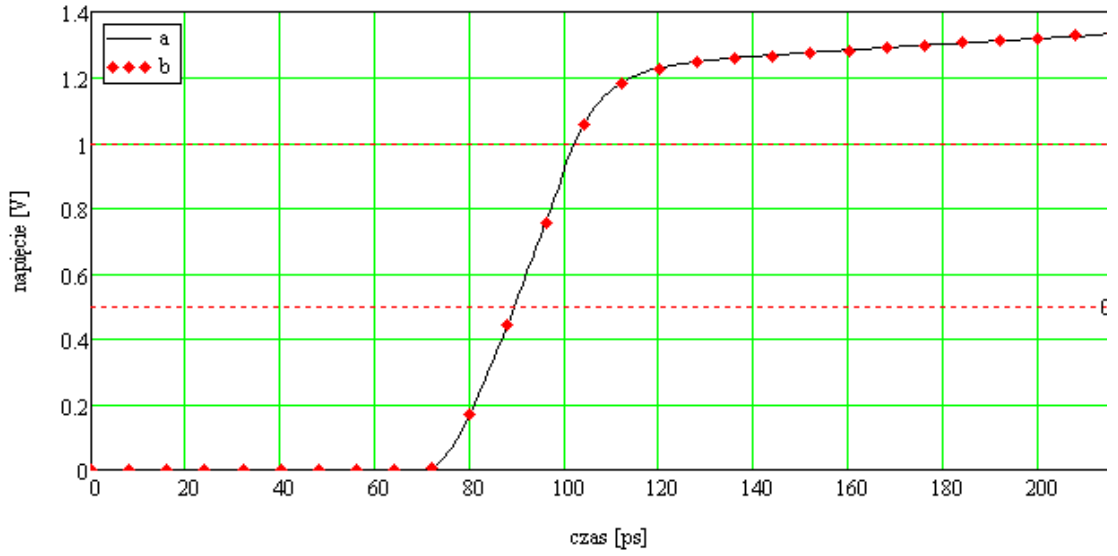
Fig. 2 Ramp response for the interconnect (Rt=25Ω, Lt=5nH, Ct=1pF, C0=0.1pF, Rw=25Ω, Tr=30ps) the output response calculated with (10) - b, compared with PSPICE simulation - a.

### Threshold crossing time calculation

The computation of threshold crossing time is a challenge many authors try to deal with, e.g. [6]. For the ramp response (10) we can calculate the threshold crossing time, during the ramp times (t<Tr) solving (11) with respect to t:

$$\rho = \frac{\left( K_1 T_2 \tilde{t} e^{-\alpha \tilde{t}} + C_2 T_r \tilde{t} - e^{-\alpha \tilde{t}} \left( K_2 T_2 - K_1 T_r e^{\alpha T_r} \right) - \frac{C_2 T_r^2}{2} - B_2 T_r \right)}{(\beta + 1)T_r}. \tag{11}$$

where $\rho$ is the threshold value (0÷1).

Equation (11) can be reduced to the form

$$T_3 = \tilde{\tau} \cdot e^{-\alpha \cdot \tilde{\tau}} + \frac{C_2 \tilde{\tau}}{K_1 T_2} - e^{-\alpha \cdot \tilde{\tau}} \cdot T_4. \tag{12}$$

and then as:

$$a_{r2} - K_r z = z e^z. \tag{13}$$

where

$a_{r2}$ and $K_r$ are constants, and z is the time function.

We can calculate the threshold crossing time using the iteration method of solving nonlinear equation $x$=f($x$), as:

$$z^{(n+1)} = W\left( a_{r2} - K_r z^{(n)} \right) \tag{14}$$

Then the threshold crossing time will take the form:

$$t_{\rho r}^{(n)} = \left( -\frac{1}{\alpha} z^{(n)} - T_4 \right) T,$$

(15)

where $z^{(n)}$ comes from (14), and for $z^{(0)}$ we take 0, what gives $z^{(1)} = W(a_{r2})$, where W(x) is Lambert W function.

### SENSITIVITY ANALYSIS

In our work we present sensitivity analysis based on the above derived, by the method of multiple scales, relationships for the voltage at output of interconnect in the system inverter – interconnect – inverter. Simulations used in investigation of the interconnect response, both the output response and threshold crossing time calculation can be highly sensitive to small changes in the parameter values. Due to generation by the presented approach the closed form formulas for output response (10), we could calculate the sensitivity of the output response with the formula:

$$S_v^{\lambda} = \frac{\lambda}{v} \frac{\partial v}{\partial \lambda}$$

(16a)

and for the threshold crossing time:

$$S_{t\rho}^{\lambda} = \frac{\lambda}{t_{\rho}} \frac{\partial t_{\rho}}{\partial \lambda}$$

(16b)

where $\lambda$ is a parameter with respect to which sensitivity is calculated.

In order to compare the results obtained analytically by means of the formulas (16) with sensitivity obtained in PSPICE we introduce the following formula using the small perturbation of parameter $\Delta\lambda$:

$$S_v^{\lambda} = \frac{\left( v(t,\lambda) - v(t,\lambda + \Delta\lambda) \right) \cdot \lambda}{v(t,\lambda) \cdot \Delta\lambda} + hot \cong$$

$$\cong \frac{v(t,\lambda) - v(t,\lambda \cdot 1.01)}{v(t,\lambda)} 100 = S_{vSPICE}^{\lambda}$$

(17)

where "hot" – means higher order terms, which has been neglected in further considerations. The small perturbation of parameter $\Delta\lambda$ is chosen for simulation as equal to 1% of $\lambda$. Simulation conducted for each of parameters confirm, that for all calculations (except for L and C, which change affect the time delay what will be analyzed later) value 1% is appropriate to obtain the proper results for sensitivity calculations. Symbol $v$ is the voltage at the end of interconnect and is denoted as $v_s$ for step response and $v_r$ for ramp response. In the paper sensitivity of the voltage with respect to set of parameters determining general features of circuit, such as perturbation parameter $\varepsilon$ responsible for losses, parameters $\alpha$, $\beta$ depending on supply and load parameters of circuit and parameters RLC of the transmission line model will be presented. Applying definition formula (16) and calculating relevant derivatives (with respect to relevant parameters) one can obtain formula in time domain for sensitivity, which may be used to calculate sensitivity in fixed time point as a function of various parameters or for fixed parameters as a function of time.

## Sensitivity analysis of the step response with respect to losses and input/output parameters

First of all we calculate sensitivity of the step response with respect to perturbation parameter ε and next we investigate influence of it on results obtained by the multiple scale method. The change of this parameter, with the other parameters unchanged, denotes the change of resistance of interconnect R on the same percentage value. Sensitivity of the step response with respect to ε is:

$$S_{v_r}^{\varepsilon} = \frac{\varepsilon}{v_r}\frac{\partial v_r}{\partial \varepsilon} = \frac{\varepsilon}{v_r}\frac{\partial}{\partial \varepsilon}\left[\frac{E_0}{(\beta+1)T_r}\left[\frac{C_2}{2}\tilde{t}^2 - B_2\tilde{t} + \left(K_1\tilde{t} - K_2\right)e^{-\alpha\tilde{t}} + K_2\right]\right]$$

$$dla \quad \tilde{t} \leq T_r/T$$

$$S_{v_r}^{\varepsilon} = \frac{\varepsilon}{v_r}\frac{\partial}{\partial \varepsilon}\frac{E_0}{(\beta+1)T_r}\left[K_1\tilde{t}e^{-\alpha\tilde{t}}T_2 + C_2\cdot T_r\tilde{t} - \left(K_2\cdot T_2 - K_1\cdot T_r e^{\alpha T_r}\right)e^{-\alpha\tilde{t}} - \frac{C_2\cdot T_r^2}{2} - I\right.$$

$$dla \quad \tilde{t} > T_r/T$$

( 18)

where $\tilde{t} = (t-T)/T$. Finally, after calculation we obtain:

$$S_{v_r}^{\varepsilon} = \frac{\varepsilon}{v_r}\frac{E_0}{(\beta+1)T_r}\left[\frac{C_{2\varepsilon}'}{2}\tilde{t}^2 - B_{2\varepsilon}'\tilde{t} + \left(K_{1\varepsilon}'\tilde{t} - K_{2\varepsilon}'\right)e^{-\alpha\tilde{t}} + K_{2\varepsilon}'\right]$$

$$dla \quad \tilde{t} \leq T_r/T$$

or

$$S_{v_r}^{\varepsilon} = \frac{\varepsilon}{v_r}\frac{E_0}{(\beta+1)T_r}\left[K_{1\varepsilon}'\tilde{t}e^{-\alpha\tilde{t}}T_2 + C_{2\varepsilon}'T_r\tilde{t} - \left(K_{2\varepsilon}'T_2 - K_{1\varepsilon}'\cdot T_r e^{\alpha T_r}\right)e^{-\alpha\tilde{t}} - \frac{C_{2\varepsilon}'T_r^2}{2} - B_{2\varepsilon}'T_r\right]$$

( 19)

$$dla \quad \tilde{t} > T_r/T$$

The sensitivity to parameter $\varepsilon$ is the same as sensitivity to the line resistance $R$. In Fig. 3 we can see comparison of results obtained by formula (19) and during PSpice simulation (using formula (17)).
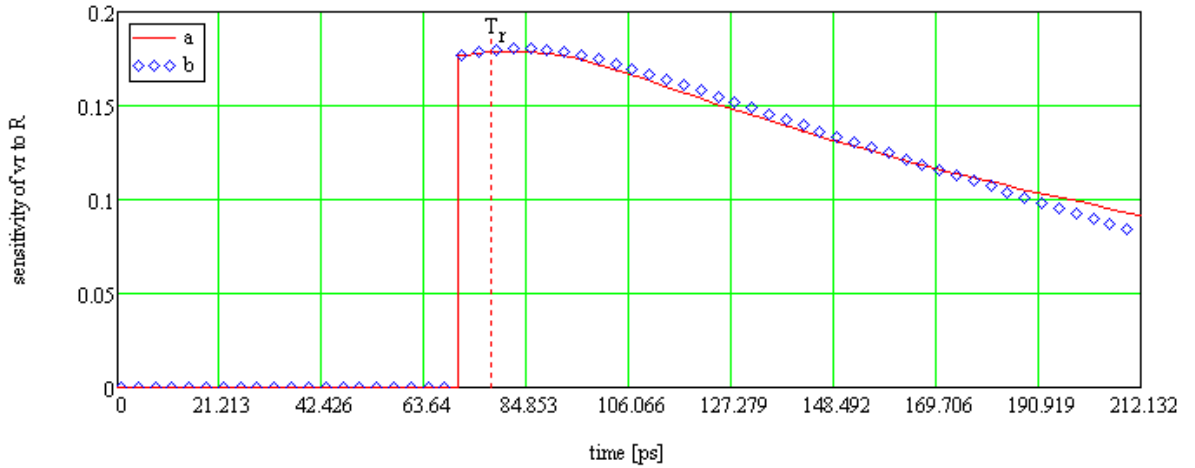
Fig. 3 Comparison of sensitivity of ramp response with respect to R calculated by formula (17) and (16), for exemplary parameters of highly inductive interconnects ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω, ⇔ $\varepsilon$=0.354, $\alpha$=10, $\beta$=0.354), a – sensitivity calculated in PSPICE, b – sensitivity calculated by means of (19).

Using relationship (19) we can show dependence of sensitivity with respect to parameter $R$ for selected sets of parameters (Fig. 4).
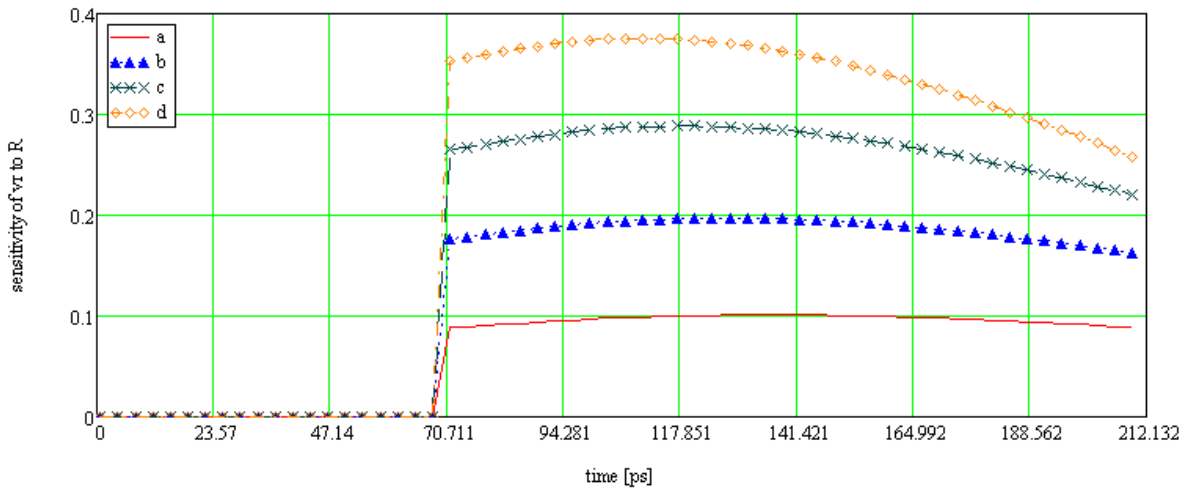


Fig. 4 Sensitivity of step response with respect to ε, as a function of time, for exemplary parameters of highly inductive interconnects ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=0.5pF and $R_w$=25Ω, ⇔ $\varepsilon$=0.354, $\alpha$=2, $\beta$=0.354) and for various values of parameter $\varepsilon$, equivalent to various values of resistance of interconnect. a – for $\varepsilon$=0.177 => $R$=12.5Ω, b – for $\varepsilon$=0.354 $R$=25Ω, c – for $\varepsilon$=0.631 => $R$=37.5Ω, d – for $\varepsilon$=0.708 => $R$=50Ω.

For early times sensitivity with respect to ε increase and for times close to 3T however decrease. In principle on sensitivity values have influence all parameters of the interconnect. The greatest sensitivities are caused by large values of R (Fig. 4), load capacitance (small α) and small values of input resistance. For considered sets of parameters the worst case value of the sensitivity 0.8 was obtained for ε = 1, α = 1, β = 0.1.

The next parameter with respect to which we investigate sensitivity of the ramp response is α containing input capacitance of the load inverter. The change of this parameter, with remaining

parameters kept constant, denotes relevant change of inverter capacitance $C_0$. The dependency is $\alpha = C/C_0$. We have:

$$\frac{dv_r}{dC_0} = \frac{\partial v_r}{\partial \alpha}\frac{\partial \alpha}{\partial C_0} + \frac{\partial v_r}{\partial \beta}\frac{\partial \beta}{\partial C_0} + \frac{\partial v_r}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial C_0} + \frac{\partial v_{r\tilde{t}}}{\partial \tilde{t}}\frac{\partial \tilde{t}}{\partial C_0}.$$

(20)

Since:

$$\frac{\partial \alpha}{\partial C_0} = -\frac{C}{C_0^2}, \qquad \frac{\partial \beta}{\partial C_0} = 0, \qquad \frac{\partial \varepsilon}{\partial C_0} = 0,$$

derivative of the ramp response has form:

$$v'_{r\,C_0} = \frac{dv_r}{dC_0} = -\frac{C}{C_0^2}\frac{\partial v_r}{\partial \alpha} = -\frac{C}{C_0^2}v'_{r\alpha}$$

and sensitivity:

$$S_{v_r}^{C_0} = -\frac{C}{C_0}\frac{v'_{r\alpha}}{v_r} = -\alpha\frac{v'_{r\alpha}}{v_r} = S_{v_r}^{\alpha}$$

(21)

Similarly to the case of perturbation parameter $\varepsilon$, we can calculate ramp response sensitivity with respect to $\alpha$ parameter from the formula:

$$S_{v_r}^{\alpha} = \frac{\alpha}{v_r}\frac{E_0}{(\beta+1)T_r}\left[\frac{C'_{2\alpha}}{2}\tilde{t}^2 - B'_{2\alpha}\tilde{t} + \left(K'_{1\alpha}\tilde{t} - K'_{2\alpha}\right)e^{-\alpha\tilde{t}} - \tilde{t}\left(K_1\tilde{t} - K_2\right)e^{-\alpha\tilde{t}} + K'_{2\alpha}\right]$$

$$dla \quad \tilde{t} \leq T_r/T$$

or

$$S_{v_r}^{\alpha} = \frac{\alpha}{v_r}\frac{E_0}{(\beta+1)T_r}\left[\begin{array}{l} K'_{1\alpha}\tilde{t}e^{-\alpha\tilde{t}}T_2 - \tilde{t}K_1\tilde{t}e^{-\alpha\tilde{t}}T_2 + K_1\tilde{t}e^{-\alpha\tilde{t}}T'_{2\alpha} + C'_{2\alpha}T_r\tilde{t} + \\ -\left(K'_{2\alpha}T_2 + K_2T'_{2\alpha} - K'_{1\alpha}T_re^{\alpha T_r} - T_rK_1T_re^{\alpha T_r}\right)e^{-\alpha\tilde{t}} + \\ +\tilde{t}\left(K_2T_2 - K_1T_re^{\alpha T_r}\right)e^{-\alpha\tilde{t}} - \frac{C'_{2\alpha}T_r^2}{2} - B'_{2\alpha}T_r \end{array}\right]$$

(22)

$$dla \quad \tilde{t} > T_r/T$$

where $A'_{2\alpha}$ $B_{2\alpha}$ $C'_{2\alpha}$ are derivatives of $A_2$, $B_2$ and $C_2$ with respect to $\alpha$.

Comparison of results from PSpice and from (22) confirms usefulness of the applied method of sensitivity calculation and the errors for the case of sensitivity to $\alpha$ are less than for the case of sensitivity to $\varepsilon$.
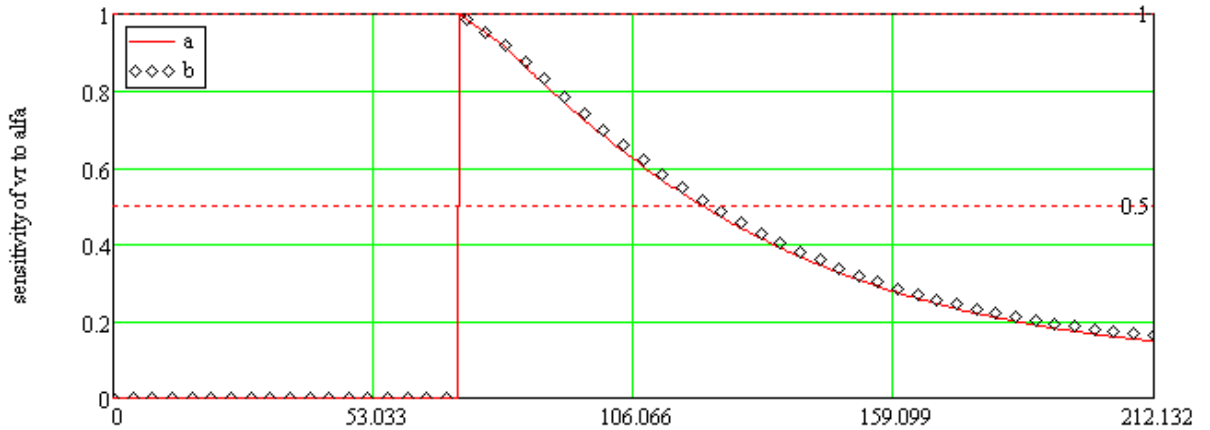
Fig. 5 Comparison of sensitivity of ramp response with respect to $\alpha$, as a function of time, from PSpice and from (22), for exemplary parameters of highly inductive interconnects ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=1pF and $R_w$=25Ω, $\Leftrightarrow \varepsilon$=0.354, $\alpha$=10, $\beta$=0.354), a – sensitivity calculated in PSpice, b – sensitivity calculated from (22).

Dependency of sensitivity to parameter $\alpha$ with respect to influence other parameters is shown in (Fig. 6). Similarly as previously the plots are presented against normalized time $\widetilde{t}$ .
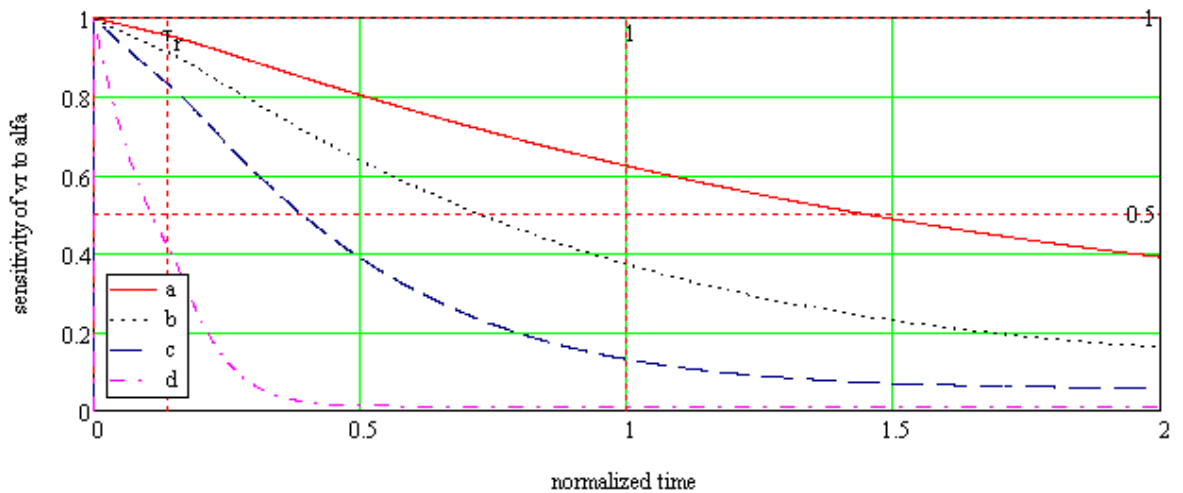


normalized time

Fig. 6 Comparison of sensitivity of ramp response with respect to $\alpha$, as a function of time, for exemplary parameters of highly inductive interconnects ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=0.5pF and $R_w$=25Ω, $\Leftrightarrow \varepsilon$=0.354, $\alpha$=2, $\beta$=0.354) for various values of parameter $\alpha$, equivalent to load capacitance changes of interconnect. a – sensitivity for parameter $\alpha$ =1 => $C_0$=1pF, b - for $\alpha$ =2 => $C_0$=0.5pF, c – for $\alpha$ =4 => $C_0$=0.25pF, d –for $\alpha$ =10 => $C_0$=0.1pF.

Sensitivity to parameter $\alpha$, in very small degree depends on changes of source resistance however strongly depends on changes of load capacitance. Sensitivity value at the moment of signal appearance at the output of interconnect is always the same and equal to one, however at the end of first wave duration is resoluteness lower (close to zero) for interconnects loaded by very small capacitances. Dependence from interconnect resistance changes (Fig. 6) is resoluteness lower than in the case of changes of the load, yet one can see tendency, that increase of resistance of interconnect ($\varepsilon$ is growing) entail increase of sensitivity.

Now we consider the sensitivity of the ramp response to parameter $\beta$ connected with the sending inverter. This parameter is proportional to the output resistance of sending inverter and can be calculated from the formula:

$$S_{v_r}^{\beta} = \frac{\beta}{v_r} \frac{E_0}{T_r} \left[ \frac{\left( \dfrac{C'_{2\beta}}{2} \tilde{t}^2 - B'_{2\beta}\tilde{t} + \left(K'_{1\beta}\tilde{t} - K'_{2\beta}\right)e^{-\alpha\tilde{t}} + K'_{2\beta} \right)}{(\beta+1)} + \right.$$

$$\left. - \frac{\left( \dfrac{C_2}{2} \tilde{t}^2 - B_2\tilde{t} + \left(K_1\tilde{t} - K_2\right)e^{-\alpha\tilde{t}} + K_2 \right)}{(\beta+1)^2} \right]$$

$$dla \quad \tilde{t} \leq T_r/T$$

(23)

$$S_{v_r}^{\beta} = \frac{\beta}{v_r} \frac{E_0}{T_r} \left[ \frac{K'_{1\beta}\tilde{t}e^{-\alpha\tilde{t}}T_2 + C'_{2\beta}T_r\tilde{t} - \left(K'_{2\beta}T_2 - K'_{1\beta}T_r e^{\alpha T_r}\right)e^{-\alpha\tilde{t}} - \dfrac{C'_{2\beta}T_r^2}{2} - B'_{2\beta}T_r}{(\beta+1)} + \right.$$

$$\left. - \frac{K_1\tilde{t}e^{-\alpha\tilde{t}}T_2 + C_2T_r\tilde{t} - \left(K_2T_2 - K_1T_r e^{\alpha T_r}\right)e^{-\alpha\tilde{t}} - \dfrac{C_2T_r^2}{2} - B_2T_r}{(\beta+1)^2} \right]$$

$$dla \quad \tilde{t} > T_r/T$$

Where $B'_{2\beta}$ and $B'_{2\beta}$ are derivatives of B and C with respect to β. The derivative of A with respect to β is zero.

Additionally basing on formula (23) we can derive sensitivity of the ramp response to $R_w$ directly and it appears that:

$$S_{v_r}^{R_w} = S_{v_r}^{\beta}.$$

(24)

Hereunder it is shown the plot, which shows exemplary waveform of the ramp response sensitivity calculated from formula (23) for typical interconnect parameters of highly inductive interconnects. For comparison we present also the waveform from the PSPICE program. One can notice, that sensitivity of the ramp response obtained on the basis of the multiple scale method, which is approximate method, differ from the results obtained from PSPICE simulation (17).
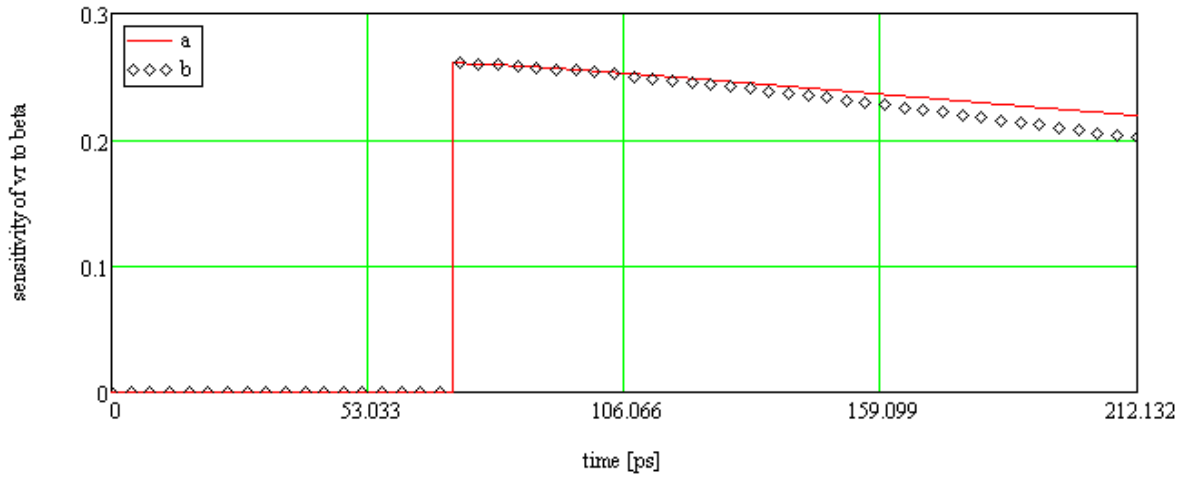
Fig. 7 Comparison of sensitivity of ramp response with respect to β, as a function of time, obtained from formula (24) and from PSPICE, for exemplary parameters of highly inductive ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=1pF and $R_w$=25Ω, ⇔ $\varepsilon$=0.354, $\alpha$=10, $\beta$=0.354), a – sensitivity calculated in PSPICE, b – sensitivity from formula (24).
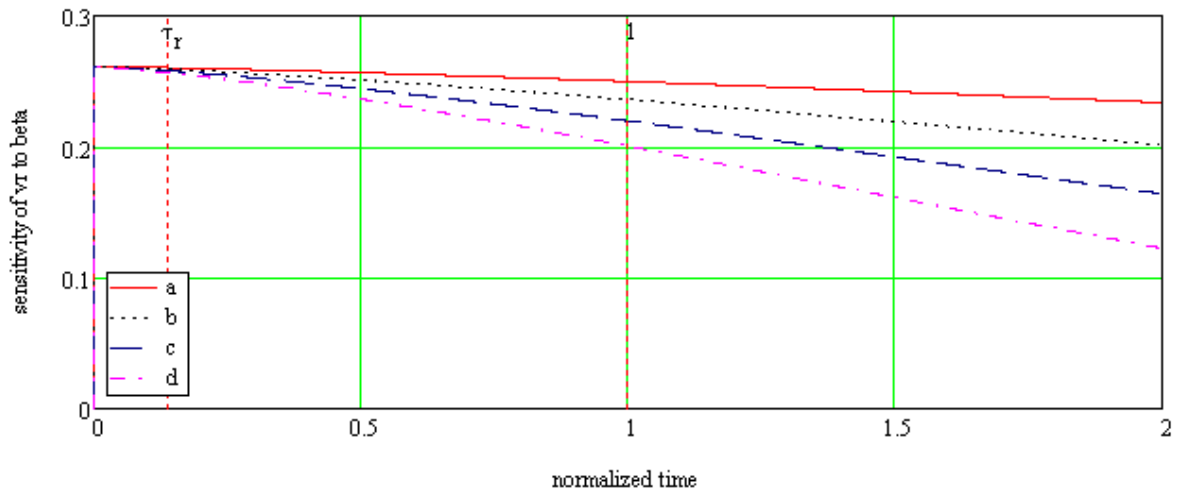


Fig. 8 Sensitivity of ramp response with respect to β, as a function of time, for exemplary parameters of highly inductive interconnects ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=0.5pF and $R_w$=25Ω, ⇔ $\varepsilon$=0.354, $\alpha$=2, $\beta$=0.354) for various values of parameter $\varepsilon$, equivalent to changes of interconnects resistance. a – for $\varepsilon$=0.177 => R=12.5Ω, b – for $\varepsilon$=0.354 R=25Ω, c – for $\varepsilon$=0.631 => R=37.5Ω, d –for $\varepsilon$=0.708 => R=50Ω.

Sensitivity to parameter β or equivalently to source resistance $R_w$, as a function of time, reach the maximal value for the time of the signal arriving to the end of interconnect depending on value of $R_w$ (β) (Fig. 8) and reach the absolute maximal value 0.41 for $R_w$=50Ω (in considered range of parameters).

## Sensitivity analysis of the ramp response with respect to C and L parameters

The problem of sensitivity to changes of RLC parameters value connects with the exactness of extraction method of interconnects parameter. Analytical formulas for calculating per unit of

length parameters RLC of interconnect are still improved.  The less the system is sensitive to small changes of parameters, the greater can be inaccuracy in performance of traces, but also in RLC parameter calculation. It is easy to show, that $S_{v_s}^R = S_{v_s}^\varepsilon$ therefore we concentrate only on sensitivity to C and L now. Sensitivity of ramp response to changes of per unit length capacitance C of interconnect using the definition of sensitivity, takes the form:

$$S_{v_r}^C = \frac{C}{v_r}\frac{\partial v_r}{\partial C} \tag{25}$$

Using directly definition, it is necessary to remember, that normalized time is function of C also. It results from this, that calculation have to be done in the not normalized time domain as follow:

$$S_{v_r}^C(\tilde{t}) = \left[\frac{1}{C_0}v_{sr}' + \frac{1}{2\sqrt{LC}}(R_w v_{r\beta}' + Rv_{rs}') - v_{r\tilde{t}}'\frac{\tilde{t}+1}{2C}\right]\frac{C}{v_r} = \tag{26}$$

$$= S_{v_r}^\alpha + \frac{1}{2}\left(S_{v_r}^\beta + S_{v_r}^\varepsilon\right) - \frac{v_{r\tilde{t}}'}{v_r}\frac{\tilde{t}+1}{2}$$

Taking into account that:

$$\tilde{t} = \frac{(t-T)}{T} = \frac{t}{T} - 1 = \frac{t}{\sqrt{L\cdot C}} - 1 \Rightarrow \tilde{t} = f(L,C) \tag{27}$$

We have:

$$\frac{dv_r}{dC} = \frac{\partial v_r}{\partial \alpha}\frac{\partial \alpha}{\partial C} + \frac{\partial v_r}{\partial \beta}\frac{\partial \beta}{\partial C} + \frac{\partial v_r}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial C} + \frac{\partial v_{r\tilde{t}}}{\partial \tilde{t}}\frac{\partial \tilde{t}}{\partial C} \tag{28}$$

and after simple but tedious calculations we arrive to formula:

$$S_{v_s}^C(t) = \frac{C}{v_s(t)}\frac{\partial v_s(t)}{\partial C} =$$

$$= \frac{C}{v_s(t)}\frac{\partial}{\partial C}\left[\frac{E_0}{\beta+1}\left(A_2\frac{t-\sqrt{LC}}{\sqrt{LC}}e^{-\alpha\frac{t-\sqrt{LC}}{\sqrt{LC}}} - B_2\cdot\left(1-e^{-\alpha\frac{t-\sqrt{LC}}{\sqrt{LC}}}\right) + C_2\frac{t-\sqrt{LC}}{\sqrt{LC}}\right)\right] \tag{29}$$

On the following plots it is presented the dependence of sensitivity of the ramp response to the per unit length capacitance C. The plot on Fig 15 (Fig 15) presents comparison sensitivity obtained in PSPICE program and from formula (29). The sensitivity in the beginning phase of time-duration of signal is very large and when $t \to T$ reaches the infinite value. In the case of using PSPICE simulations and formula (16) the error for time close to T is very large, therefore we take constant sensitivity equal to $1/\Delta\lambda$ for the time range from T to $T = \sqrt{LC(1+\Delta\lambda)}$ .
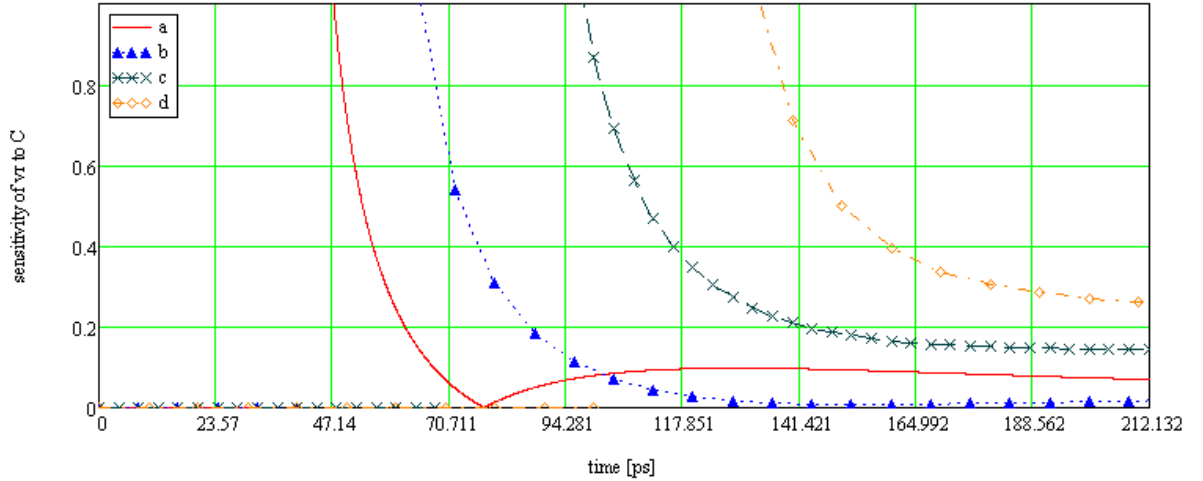
Fig. 9 Sensitivity of ramp response to per unit length capacitance of interconnect, for exemplary parameters of highly inductive interconnects ($R_t$=25Ω, $L_t$=5nH, $C_t$=1pF, $C_0$=0.5pF and $R_w$=25Ω, $\Leftrightarrow$ $\varepsilon$=0.354, $\alpha$=2, $\beta$=0.354) for various values of interconnect capacitance $C$. a – for $C$=1pF, b – for $C$=0.5pF, c – for $C$=0.2pF, d – for $C$=0.1pF.

Inductance of interconnect understood as an inductance per unit length of transmission line modeling interconnect is for top level interconnects very important factor, neglecting of which can cause large errors in simulation. Inductance significantly affects on signal integrity, hence influence of inductance on sensitivity of ramp response is very important element of top level layers of interconnects analysis. Sensitivity of ramp response to inductance, according to definition, can be written in the following form:

$$S_{v_r}^L = \frac{L}{v_r}\frac{\partial v_r}{\partial L} \tag{30}$$

Similarly as previously it is yet possible to exploit previously derived relationships on sensitivity to parameters $\alpha$, $\beta$, $\varepsilon$, for normalized time as:

$$\frac{dv_r}{dL} = \frac{\partial v_r}{\partial \alpha}\frac{\partial \alpha}{\partial L} + \frac{\partial v_r}{\partial \beta}\frac{\partial \beta}{\partial L} + \frac{\partial v_r}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial L} + \frac{\partial v_{r\tilde{t}}}{\partial \tilde{t}}\frac{\partial \tilde{t}}{\partial L} \tag{31}$$

For not normalized time we obtain:

$$S_{v_r}^L(t) = -\frac{1}{2}\left(S_{v_r}^\beta(t) + S_{v_r}^\varepsilon(t)\right) - \frac{v'_{r\tilde{t}}}{v_r}\frac{t}{2\sqrt{LC}} \tag{32}$$

where $v'_{r\tilde{t}}$ is a derivative of $v_r$ to normalized time $\tilde{t}$.

Sensitivity calculated analytically (32) and in PSPICE program are in good agreement. To calculate sensitivity in PSPICE program we use formula (17), therefore at the moment $t$=$T$ we will

designate it value $1/\Delta\lambda$. However in fact at $t=T$ it takes infinite value (Fig. 9). This improper value of sensitivity is designated in the time range $\sqrt{LC} < t < \sqrt{LC(1 + d\lambda)}$.

On the following plots we present sensitivity of ramp response to interconnect inductance for various parameters of inverter-interconnect-inverter system as: $R$, $L$, $C$, $R_w$, $C_0$.
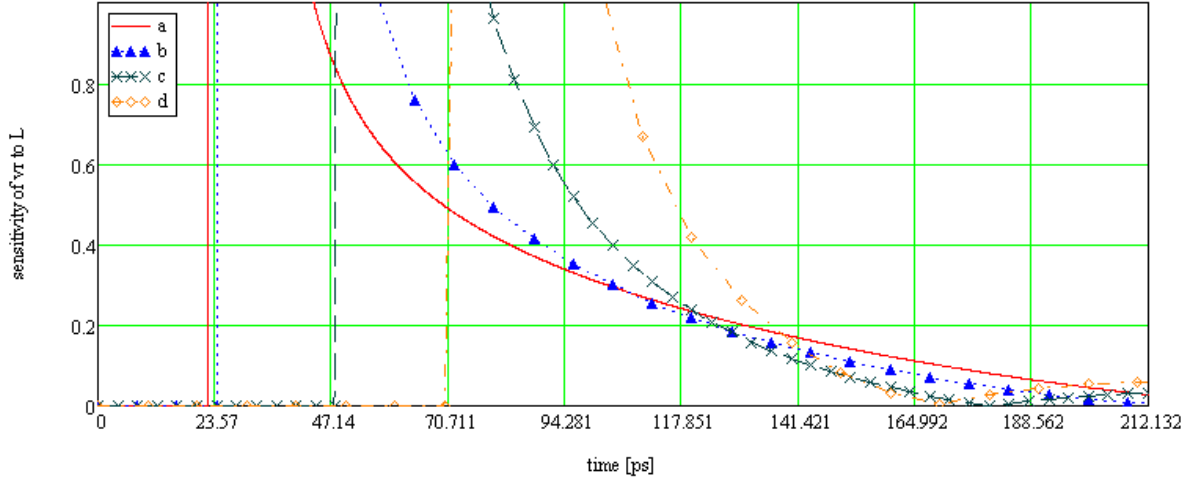


Fig. 10 Sensitivity of ramp response to per unit length inductance of interconnect, for exemplary parameters of highly inductive ($R_t$=25$\Omega$, $L_t$=5nH, $C_t$=1pF, $C_0$=0.5pF and $R_w$=25$\Omega$, $\Leftrightarrow \varepsilon$=0.354, $\alpha$=2, $\beta$=0.354) for various values of interconnect capacitance $C$. a – for $C$=1pF, b – for $C$=0.5pF, c – for $C$=0.2pF, d – for $C$=0.1pF.

Analysis of ramp response, for which exemplary plots has been presented above, permits to formulate the conclusion, that sensitivity of ramp response to interconnect inductance for early times is very large, yet after time 2T (from the beginning of the time axis) is, in considered here range of parameters, less that one. Very large values of sensitivity for times close to T are result of dependence of T on inductance L (for $t=T$ sensitivity achieves value infinity). As time is going down, time delay caused by change of inductance has no such meaning. In this connection it seems very important, form the point of view of threshold crossing time calculation, correct calculation of interconnect inductance.

## Sensitivity of threshold crossing time of ramp response

Sensitivity of crossing time over voltage threshold to interconnect parameters is equivalent to relative change of the threshold crossing time to a change of parameter. In this section we present the sensitivity analysis of threshold crossing time to per unit length interconnect parameters $R$ $L$ $C$ and to parameters connected with load and supply also. Time of crossing over voltage threshold, what was shown above, can be determined on the basis of step or/and ramp responses for majority of cases of low loss and highly inductive interconnects in the time range $T_r<t<3T$. In this connection the threshold value we calculate from formula. Sensitivity of threshold crossing time to parameter $\lambda$ can be defined:

$$S_{t_{pr}}^{\lambda} = \frac{\partial t_{pr}}{\partial \lambda} \frac{\lambda}{t_{pr}}$$

(33)

In general case, when it is essential to use iterative formula for threshold crossing time (15), calculation of sensitivity from formula directly is impossible. Thus it is necessary to create the

method, which enable sensitivity calculation on the basis of threshold crossing time relationship. The proposed method consist in differentiation of the both sides of equation:

$$\rho(\beta+1)\tau_r = K_1\tau e^{-\alpha\tau}T_2 + C_2\tau_r\tau - e^{-\alpha\tau}(K_2T_2 - K_1\tau_r e^{\alpha\tau_r}) - \frac{1}{2}C_2\tau_r^2 - B_2\tau_r \qquad (34)$$

and next calculation of sensitivity. Detailed formulas and derivations will be shown in the next sections.

## Sensitivity to interconnect model parameters

We start our calculation of sensitivity with respect to following parameters: perturbation parameter $\varepsilon$, load parameter $\alpha$, supply parameter $\beta$ and $T_r$. Calculations of sensitivity to $\varepsilon$ require differentiation of (34) with respect to $\varepsilon$ taking into account, that $\tilde{t}_{pr} = (t_{pr} - \sqrt{LC})/\sqrt{LC}$ depends on $\varepsilon$. After differentiation (34) we obtain:

$$0 = \frac{\partial K_1}{\partial \varepsilon}\tilde{t}_{pr}e^{-\alpha\tilde{t}_{pr}}T_2 + K_1\frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}e^{-\alpha\tilde{t}_{pr}}T_2 + K_1\tilde{t}_{pr}e^{-\alpha\tilde{t}_{pr}}(-\alpha)\frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}T_2 + \frac{\partial C_2}{\partial \varepsilon}\tau_r\tilde{t}_{pr} + C_2\tau_r\frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}$$
$$+ \alpha e^{-\alpha\tilde{t}_{pr}}\frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}(K_2T_2 - K_1\tau_r e^{\alpha\tau_r}) - e^{-\alpha\tilde{t}_{pr}}\left(\frac{\partial K_2}{\partial \varepsilon}T_2 - \frac{\partial K_1}{\partial \varepsilon}\tau_r e^{\alpha\tau_r}\right) - \frac{1}{2}\frac{\partial C_2}{\partial \varepsilon}\tau$$
$$- \frac{\partial B_2}{\partial \varepsilon}\tau_r \qquad \text{35)}$$

Denoting $\tau'_{pr\varepsilon} = \frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}$ and solving the above equation with respect to $\frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}$ we have:

$$\tau'_{pr\varepsilon} = \frac{e^{-\alpha\tilde{t}_{pr}}\left(\frac{\partial K_2}{\partial \varepsilon}T_2 - \frac{\partial K_1}{\partial \varepsilon}\tau_r e^{\alpha\tau_r} - \frac{\partial K_1}{\partial \varepsilon}\tilde{t}_{pr}T_2\right) - \frac{\partial C_2}{\partial \varepsilon}\tau_r\tilde{t}_{pr} + \frac{1}{2}\frac{\partial C_2}{\partial \varepsilon}\tau_r^2 + \frac{\partial B_2}{\partial \varepsilon}\tau_r}{K_1e^{-\alpha\tilde{t}_{pr}}T_2 + K_1\tilde{t}_{pr}e^{-\alpha\tilde{t}_{pr}}(-\alpha)T_2 + C_2\tau_r + \alpha e^{-\alpha\tilde{t}_{pr}}(K_2T_2 - K_1\tau_r e^{\alpha\tau_r})} \qquad (36)$$

where derivative $\frac{A_2}{\partial \varepsilon}, \frac{B_2}{\partial \varepsilon}, \frac{\partial C_2}{\partial \varepsilon}$ could be easily obtained.

Sensitivity to $\varepsilon$ will be described by formula:

$$S^{\varepsilon}_{t_{pr}} = \tilde{t}'_{pr\varepsilon}\frac{\varepsilon}{\tilde{t}_{pr}} \qquad (37)$$

It is easy to show, that sensitivity of ramp response to $\varepsilon$ is equivalent to sensitivity to interconnect resistance R. Finally we have:

$$S^{R}_{t_{pr}} = \frac{1}{Z_0}t'_{pr\varepsilon}\frac{R}{t_{pr}} = \frac{\varepsilon}{t_{pr}}t'_{pr\varepsilon} = S^{\varepsilon}_{t_{pr}} \qquad (38)$$

The following plots present relationships of sensitivity of 50% threshold crossing time to perturbation parameter $\varepsilon$ (resistivity of interconnect) for exemplary interconnect parameters.
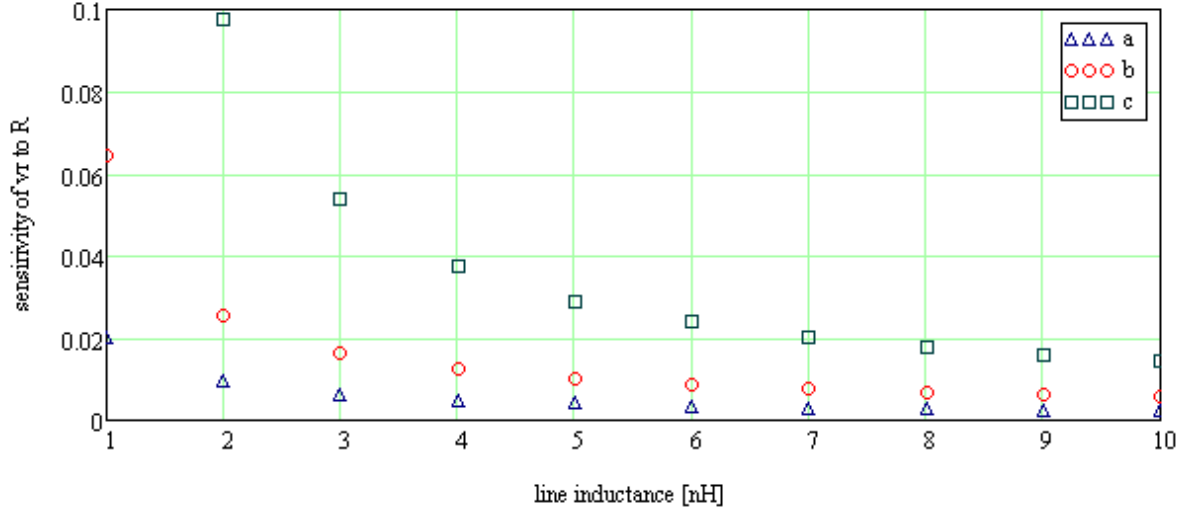
Fig. 11 Sensitivity of 50% threshold crossing time to perturbation parameter ε (37), as a function of interconnect inductance L, for various values of interconnect resistance. Sensitivity has been drawn for parameters: a - $R_t$=12.5Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω, b - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω, c - $R_t$=50Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω.

The above analysis shows, that sensitivity of 50% threshold crossing time to interconnect resistance depends, in very small degree, on load capacitance and rise time of the input ramp signal. The largest influence on sensitivity has interconnect inductance and load capacitance. Large values of interconnect capacitance in connection with small interconnect inductance cause the largest sensitivity to interconnect resistance. Larger sensitivity have more lossy interconnects, similar influence we can notice when interconnect is supplied from inverter with larger output resistance. Sensitivity analysis for interconnect inductance changes, permits to come to the conclusion, that larger inductance the smaller sensitivity to interconnect losses determined by parameter ε.

Sensitivity of crossing time threshold of the ramp signal to parameter β can be executed by differentiation of (34) with respect to β:

$$\rho\tau_r = K_1 \frac{\partial \tilde{t}_{\rho r}}{\partial \beta} e^{-\alpha \check{t}_{\rho r}} T_2 + K_1 \tilde{t}_{\rho r}(-\alpha) \frac{\partial \tilde{t}_{\rho r}}{\partial \beta} e^{-\alpha \check{t}_{\rho r}} T_2 + \frac{\partial C_2}{\partial \beta} \tau_r \tilde{t}_{\rho r} + C_2 \tau_r \frac{\partial \tilde{t}_{\rho r}}{\partial \beta}$$
$$+ \alpha e^{-\alpha \check{t}_{\rho r}} \frac{\partial \tilde{t}_{\rho r}}{\partial \beta} (K_2 T_2 - K_1 \tau_r e^{\alpha \tau_r}) - e^{-\alpha \check{t}_{\rho r}} \left( \frac{\partial K_2}{\partial \beta} T_2 - 0 \right)$$
$$- \frac{1}{2} \frac{\partial C_2}{\partial \beta} \tau_r^2 - \frac{\partial B_2}{\partial \beta} \tau_r$$

39)

derivative of crossing time over voltage threshold is determined by formula

$$\tau'_{\rho r \beta} = \frac{\rho \tau_r - \frac{\partial C_2}{\partial \beta} \tau_r \tilde{t}_{\rho r} + e^{-\alpha \check{t}_{\rho r}} \frac{\partial K_2}{\partial \beta} T_2 + \frac{1}{2} \frac{\partial C_2}{\partial \beta} \tau_r^2 + \frac{\partial B_2}{\partial \beta} \tau_r}{K_1 e^{-\alpha \check{t}_{\rho r}} T_2 - K_1 \tilde{t}_{\rho r} \alpha e^{-\alpha \check{t}_{\rho r}} T_2 + C_2 \tau_r + \alpha e^{-\alpha \check{t}_{\rho r}} (K_2 T_2 - K_1 \tau_r e^{\alpha \tau_r})}$$

( 40)

where

$$\frac{\partial K_2}{\partial \beta} = \frac{\partial \left( \frac{B_2}{\alpha} + \frac{A_2}{\alpha^2} \right)}{\partial \beta} = \frac{1}{\alpha} \frac{\partial B_2}{\partial \beta},$$

knowing (40) we can express sensitivity to β as follows:

$$S^{\beta}_{t_{pr}} = \tilde{t}'_{pr\beta} \frac{\beta}{t_{pr}}$$

(41)

Basing on sensitivity to β we can derive, like previously, sensitivity to source resistance $R_w$. After some transformations we arrive to relationship:

$$S^{R_w}_{\tilde{t}_{pr}} = \frac{1}{Z_0} \tilde{t}'_{pr\beta} \frac{R_w}{\tilde{t}_{pr}} = \frac{\beta}{\tilde{t}_{pr}} \tilde{t}'_{pr\beta} = S^{\beta}_{\tilde{t}_{pr}}$$

(42)

Obtained relationships permits for obtaining values of sensitivities to output resistance of sending inverter for concrete interconnect parameters. The results has been compared with results of simulation in PSPICE program. Exemplary values of parameters are presented on a plot in Fig. 12.
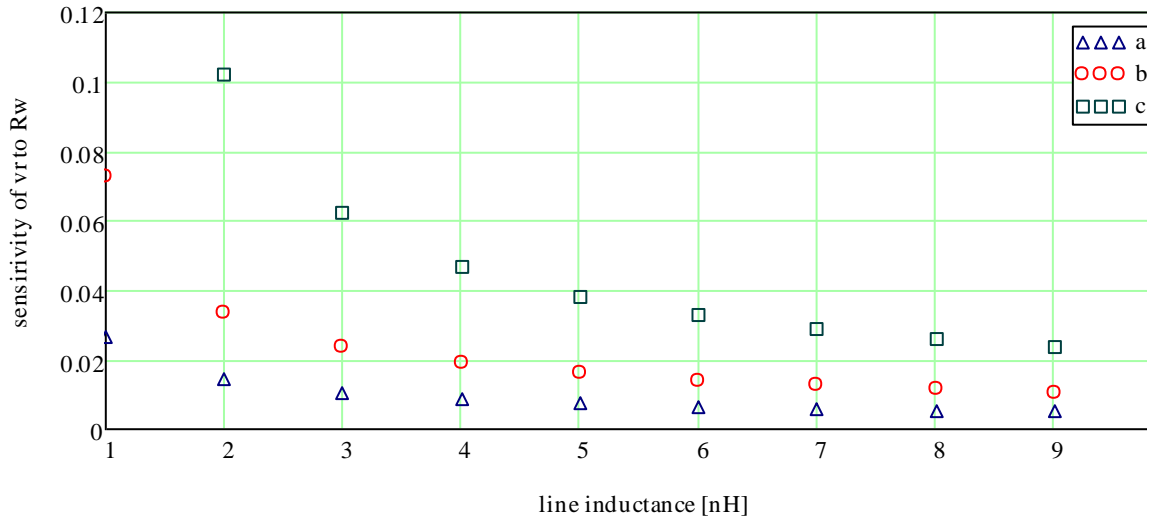


Fig. 12 Sensitivity of 50% threshold crossing time to parameter Rw (42), as a function of interconnect inductance L for 3 values of interconnect resistance. Sensitivity has been drawn for parameters: a - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=12.5Ω, b - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω, c - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=50Ω.

Simulations performed in order to make conclusions on influence interconnects model parameters, in particular interconnect inductance, on sensitivity crossing time over given voltage threshold, confirm, that the larger interconnect inductance and smaller resistance the smaller sensitivity to output resistance of sending inverter. One can also to notice, that rise time of ramp signal and interconnect capacitance have not large influence, if parameters fulfill conditions of lowloss interconnect. On the plot we have shown exemplary values of sensitivity to output resistance of sending inverter for selected interconnect parameters, which illustrate range of changes and tendency of sensitivity to changes of parameter $R_w$.

Sensitivity crossing time over voltage threshold to load capacitance, will be calculated from sensitivity to $\alpha$ parameter:

$$S^{\alpha}_{t_{pr}} = \tilde{t}'_{pr\alpha} \frac{\alpha}{t_{pr}}$$

(43)

$$\tilde{t}'_{prRC_0} = \frac{d\tilde{t}_{pr}}{dC_0} = -\frac{C}{C_0^2} \frac{\partial \tilde{t}_{pr}}{\partial \alpha} = -\frac{C}{C_0^2} \tilde{t}'_{pr\alpha}$$

$$S_{\tilde{t}_{pr}}^{C_0} = -\frac{C}{C_0^2}\tilde{t}'_{pr\alpha}\frac{C_0}{\tilde{t}_{pr}} = \frac{\alpha}{\tilde{t}_{pr}}\tilde{t}'_{pr\alpha} = S_{\tilde{t}_{pr}}^{\alpha}$$

<div align="right">(44)</div>

where $S_{\tilde{t}_{pr}}^{\alpha}$ can be determined by differentiation of equation (34) and next solving it for derivative of crossing time over voltage threshold with respect to α. After differentiation of (34) we have:

$$0 = \frac{\partial K_1}{\partial \alpha}\tilde{t}_{pr}e^{-\alpha\tilde{t}_{pr}}T_2 + K_1\frac{\partial\tilde{t}_{pr}}{\partial\alpha}e^{-\alpha\tilde{t}_{pr}}T_2 - \alpha K_1\tilde{t}_{pr}^2 e^{-\alpha\tilde{t}_{pr}}T_2 - K_1\tilde{t}_{pr}e^{-\alpha\tilde{t}_{pr}}\frac{\partial\tilde{t}_{pr}}{\partial\alpha}T_2$$
$$+ K_1\tilde{t}_{pr}e^{-\alpha\tilde{t}_{pr}}\frac{\partial T_2}{\partial\alpha} + C_2\tau_r\frac{\partial\tilde{t}_{pr}}{\partial\alpha} + e^{-\alpha\tilde{t}_{pr}}\tilde{t}_{pr}(K_2T_2 - K_1\tau_r e^{\alpha\tau_r})$$
$$+ e^{-\alpha\tilde{t}_{pr}}\alpha\frac{\partial\tilde{t}_{pr}}{\partial\alpha}(K_2T_2 - K_1\tau_r e^{\alpha\tau_r})$$
$$- e^{-\alpha\tilde{t}_{pr}}\left(\frac{\partial K_2}{\partial\alpha}T_2 + K_2\frac{\partial T_2}{\partial\alpha} - \frac{\partial K_1}{\partial\alpha}\tau_r e^{\alpha\tau_r} - K_1\tau_r^2 e^{\alpha\tau_r}\right) - \frac{\partial B_2}{\partial\alpha}\tau_r$$

<div align="right">45)</div>

Derivative $\tilde{t}_{pr}$ with respect to α is expressed by formula:

$$\tilde{t}'_{pr\alpha} = \frac{e^{-\alpha\tilde{t}_{pr}}\left(\frac{\partial K_2}{\partial\alpha}T_2 + K_2\frac{\partial T_2}{\partial\alpha} - \frac{\partial K_1}{\partial\alpha}\tau_r e^{\alpha\tau_r} - K_1\tau_r^2 e^{\alpha\tau_r} - \frac{\partial K_1}{\partial\alpha}\tilde{t}_{pr}T_2\right)}{e^{-\alpha\tilde{t}_{pr}}\left(\alpha(K_2T_2 - K_1\tau_r e^{\alpha\tau_r}) + K_1T_2(1 - \alpha K_1\tilde{t}_{pr})\right) + C_2\tilde{t}_{pr\tau_r}}$$
$$+ \frac{e^{-\alpha\tilde{t}_{pr}}\left(K_1\tilde{t}_{pr}^2 T_2 - K_1\tilde{t}_{pr}\frac{\partial T_2}{\partial\alpha} - \tilde{t}_{pr}(K_2T_2 - K_1\tau_r e^{\alpha\tau_r})\right) + \frac{\partial B_2}{\partial\alpha}\tau_r}{e^{-\alpha\tilde{t}_{pr}}\left(\alpha(K_2T_2 - K_1\tau_r e^{\alpha\tau_r}) + K_1T_2(1 - \alpha K_1\tilde{t}_{pr})\right) + C_2\tilde{t}_{pr\tau_r}}$$

<div align="right">46)</div>

where

$$\frac{\partial K_1}{\partial\alpha} = \frac{A_2}{\alpha^2}$$
$$\frac{\partial T_2}{\partial\alpha} = -\tau_r e^{\alpha\tau_r}$$
$$\frac{\partial K_2}{\partial\alpha} = \frac{\frac{\partial B_2}{\partial\alpha}\alpha - B_2}{\alpha^2} + \frac{-A_2 2\alpha}{\alpha^4} = \frac{\frac{\partial B_2}{\partial\alpha}\alpha - B_2}{\alpha^2} - \frac{2A_2}{\alpha^3}$$

Sensitivity to load capacitance for considered range of interconnect parameters is the largest for large values of load capacitance and its maximal value is 0.5. Significant influence on increasing of sensitivity have also large values of resistance, and small values of capacitance and inductance of interconnect, and large values of source resistance. In Fig. 13 is shown exemplary results for sensitivity to load capacitance (receiving inverter input capacitance) calculation for selected parameters illustrating influence of values of these parameters on sensitivity.
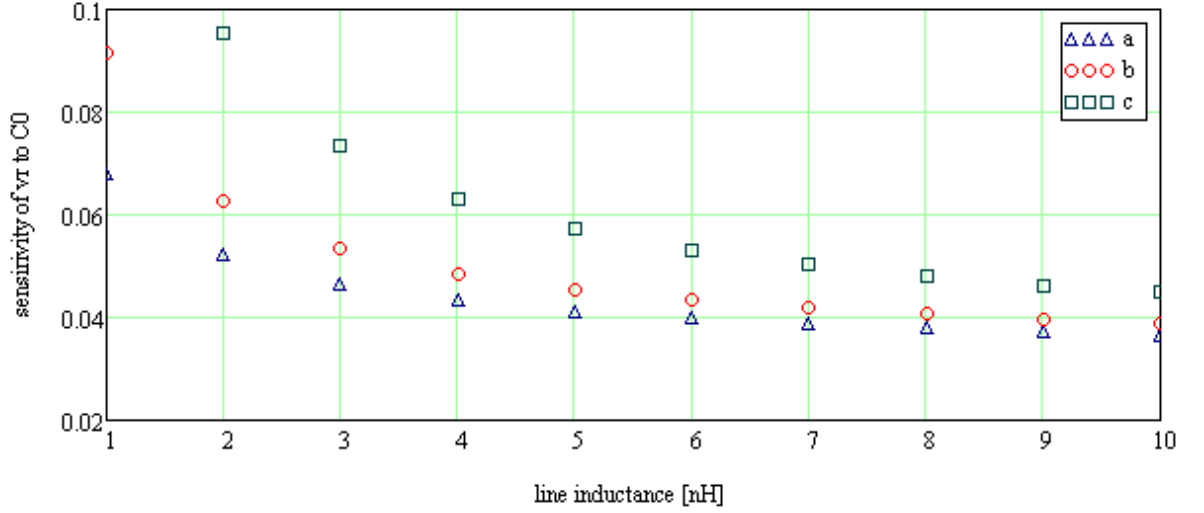
Fig. 13 Sensitivity of 50% threshold crossing time to load capacitance $C_0$ (44), as a function of interconnect inductance L for 3 values of interconnect resistance. Sensitivity has been drawn for parameters: a - $R_t$=12.5$\Omega$, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25$\Omega$, b - $R_t$=25$\Omega$, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25$\Omega$, c - $R_t$=50$\Omega$, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25$\Omega$.

Sensitivity of threshold crossing time to the last of parameters determining properties of input/output, that is sensitivity to rise-time of the ramp signal can be calculate on the basis derivative of threshold crossing time with respect to variable the rise-time. Differentiation of the both sides of (34) give us:

$$\rho(\beta+1) = K_1 \frac{\partial \tilde{t}_{pr}}{\partial \tau_r} e^{-\alpha \tilde{t}_{pr}} T_2 + K_1 \tilde{t}_{pr}(-\alpha) \frac{\partial \tilde{t}_{pr}}{\partial \tau_r} e^{-\alpha \tilde{t}_{pr}} T_2 + K_1 \tilde{t}_{pr} e^{-\alpha \tilde{t}_{pr}} \frac{\partial T_2}{\partial \tau_r} + C_2 \tilde{t}_{pr} +$$

$$+ \alpha e^{-\alpha \tilde{t}_{pr}} \frac{\partial \tilde{t}_{pr}}{\partial \tau_r} (K_2 T_2 - K_1 \tau_r e^{\alpha \tau_r}) - e^{-\alpha \tilde{t}_{pr}} \left( K_2 \frac{\partial T_2}{\partial \tau_r} - K_1 e^{\alpha \tau_r} - K_1 \tau_r \alpha \right.$$

$$\left. - \frac{1}{2} C_2 2\tau_r - B_2 \right.$$

47)

Solving the above equation with respect to derivative, we obtain:

$$\tilde{t}'_{pr\tau_r} = \frac{\rho(\beta+1) + C_2(\tau_r - \tilde{t}_{pr}) + B_2 + e^{-\alpha \tilde{t}_{pr}} e^{\alpha \tau_r} (A_2(\tau_r - \tilde{t}_{pr}) - B_2)}{C_2 \tau_r + e^{-\alpha \tilde{t}_{pr}} (B_2 T_2 - K_1 \tilde{t}_{pr} \alpha T_2 - \alpha K_1 \tau_r e^{\alpha \tau_r})}$$

(48)

and next sought relationship for sensitivity in the form:

$$S_{t_{pr}}^{\tau_r} = \tilde{t}'_{pr\tau_r} \frac{\tau_r}{t_{pr}}$$

(49)

Sensitivity to rise-time of input ramp signal (49) in small degree depend on model of inverter-interconnect-inverter system parameters, particularly to the changes of interconnect resistance $R$

and output resistance of sending inverter $R_w$. Along with growth of inductance sensitivity value diminishes, similarly for changes of interconnect capacitance $C$ and load capacitance $C_0$ (Fig. 14). The strongest dependence of sensitivity is for changes of the rise-time, and for changes of the rise-time equal 70% the time delay T can reach value 0.6, while for 10% for the same interconnect sensitivity is equal to 0.15.
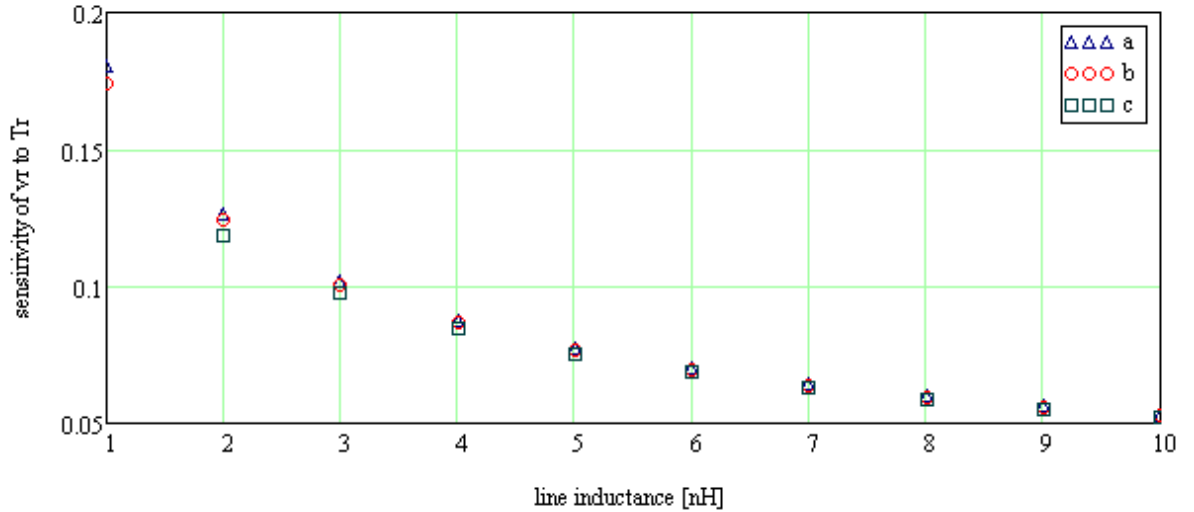


Fig. 14 Sensitivity of 50% threshold crossing time to rise-time input ramp signal (49), as a function of interconnect inductance L for 3 values of sending inverter output resistance. Sensitivity has been drawn for parameters: a - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=12.5Ω, b - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω, c - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=50Ω.

We can calculate sensitivity of threshold crossing time to interconnect inductance $L$:

$$\frac{d\tilde{t}_{pr}}{dL} = \frac{\partial \tilde{t}_{pr}}{\partial \alpha}\frac{\partial \alpha}{\partial L} + \frac{\partial \tilde{t}_{pr}}{\partial \beta}\frac{\partial \beta}{\partial L} + \frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial L} \qquad (50)$$

where:

$$\frac{\partial \alpha}{\partial L} = 0, \qquad \frac{\partial \varepsilon}{\partial L} = -\frac{1}{2}\frac{R}{L}\sqrt{\frac{C}{L}}, \qquad \frac{\partial \beta}{\partial L} = -\frac{1}{2}\frac{R_w}{L}\sqrt{\frac{C}{L}}$$

in turn $\frac{\partial \tilde{t}_{pr}}{\partial \alpha}, \frac{\partial \tilde{t}_{pr}}{\partial \beta}, \frac{\partial \tilde{t}_{pr}}{\partial \varepsilon}$ can be easily calculated.

On the plot is presented comparison of the results obtained by the analytical approach determining sensitivity of threshold crossing time to interconnect inductance with simulation done in program PSPICE. The results of analysis of sensitivity for various parameter values permits to say, that the largest influence on sensitivity value have load capacitance. Since significance of interconnect inductance on behavior of inverter-interconnect-inverter system sensitivity is the main problem of this paper.

On the following plot we present sensitivity for various sets of interconnect parameter values to illustrate the influence of these values on sensitivity.
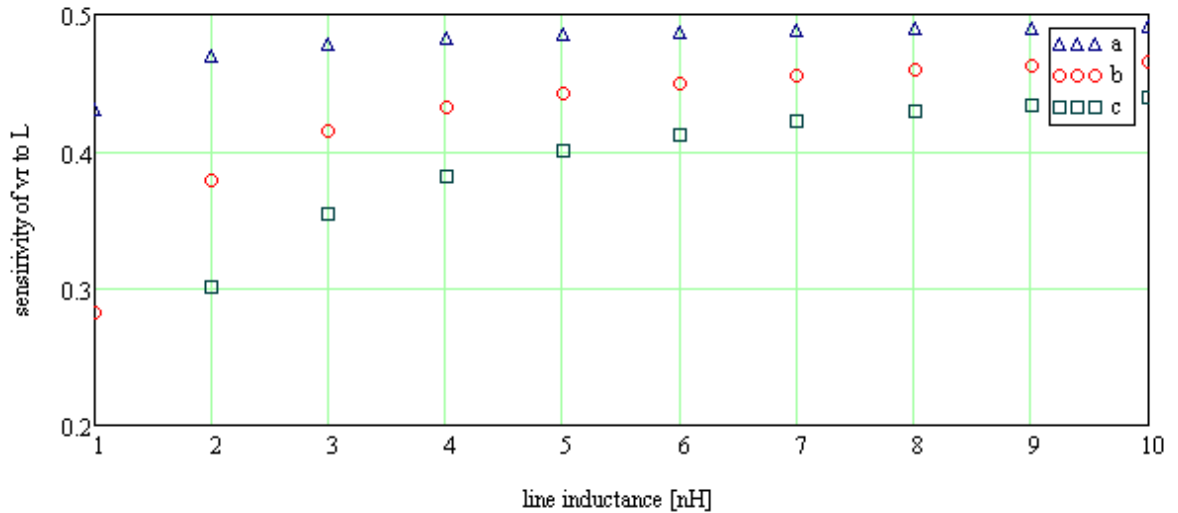
Fig. 15 Sensitivity of 50% threshold crossing time to interconnect inductance $L$, as a function of interconnect inductance L for 3 values of load capacitance $C_0$. Sensitivity has been drawn for parameters: a - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.1pF and $R_w$=25Ω, b - $R_t$=25Ω, $C_t$=1pF, $C_0$=0.5pF and $R_w$=25Ω, c - $R_t$=25Ω, $C_t$=1pF, $C_0$=1pF and $R_w$=25Ω.

## CONCLUSIONS

Analysis of sensitivity values to various parameters permits for general conclusions formulation. Sensitivity to the most of parameters decreases as inductance increase and resistance decrease. The large load capacitance means larger sensitivity to the most parameters too. The derived relationships as sensitivity functions permits for fast calculation of sensitivities for specific interconnect parameters, without carry out arduous response simulations and next, basing on them, using incremental method of analysis to calculate sensitivity. The presented analytical method, which exactness was confirmed by simulations, has application in analysis of interconnect of higher layers VLSI circuits, assuming, that losses of interconnect are small and can be treated as a small perturbation parameter.

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