SENSITIVITY ANALYSIS OF VLSI INTERCONNECT OUTPUT SIGNAL

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ABSTRACT: Interconnect parameters play an important role in signal propagation in VLSI systems. In the paper we present the sensitivity analysis to parameters of typical on-chip interconnects for the step and ramp response. In the paper we show also the sensitivity to parameters for the threshold crossing time. We compute the output response of the interconnect and the threshold crossing time formulas based on the multiple scales method.

INTRODUCTION

Faster circuits, higher frequencies, smaller die sizes in VLSI modern technology causes new problems in system designs. During that process there is a need to appropriate model assumption to ensure that the simulation of the system will give good accuracy. Both the gates and interconnect must be considered during system modeling. Due to the resistance of the modern interconnects are smaller than the lossless line impedance Z_0 (1) there is no longer possible to model the interconnect by RC transmission line, and the RLC transmission line must be considered [1,4].

$$\frac{R}{2Z_0} \le 1 \tag{1}$$

Although the high growth of device speeds the on-chip interconnect has not been scaling so fast. Especially global and clock wires could be 2-8x the minimum dimensions [1]. The higher level clock interconnects might have total resistance 8.2-21.5 Ω for 1mm line length. Most of the application for simulation use only RC model, and that approach does not give good accuracy [1]. Typical simulation problems consider simulation of one interconnect or more coupled interconnects.

In our work we derive the method for compute the step and ramp responses for clock and global wires assuming RLC transmission model and the global resistance less than lossless interconnect impedance. That assumption allows treat the line as low loss line with high inductance influence. For the system of differential equation (2) with initial and boundary conditions (3,4) we could use the perturbation method of multiple scales, using the $\varepsilon = \frac{R_t}{Z_0}$ as perturbation parameter ($R_t = R \cdot d$).

$$\begin{cases} -\dfrac{\partial v}{\partial x} = Ri + L\dfrac{\partial i}{\partial t}, \\ -\dfrac{\partial i}{\partial x} = C\dfrac{\partial v}{\partial t} \end{cases} \tag{2}$$

$$i(x,0) = 0, \quad v(x,0) = 0, \tag{3}$$

$$\begin{aligned} &e(t) - R_s i(0,t) = v(0,t), \\ &-i(d,t) = C_0 \frac{\partial v(d,t)}{\partial t}, \end{aligned} \tag{4}$$

where

R, L, C – line parameters, C0 – input inverter capacitance, Rs – output inverter resistance, i(x,t), v(x,t) – current and voltage in line, respectively, d – line length, t, x – time and space variable, respectively.

After scaling system (2) to obtain the perturbation parameter in the system, we can observe, that resistance of the interconnect influence only for perturbation parameter value. After scaling we have:

$$\begin{aligned} &y = \frac{x}{d}, \ \tau = \frac{t}{\sqrt{L_t C_t}}, \ \tilde{v} = -\sqrt{\frac{C_t}{L_t}} v, \\ &\tilde{e} = \sqrt{\frac{C_t}{L_t}} e, \ \beta = \sqrt{\frac{C_t}{L_t}} R_s \end{aligned} \tag{5}$$

Then calculating the system with multiple scales method we calculate the lossless interconnect what is typical task and than extend the solution with perturbation parameter.

The paper is organized as follows: in the second section there we shortly present the method of step and ramp response calculation. In the third section we show how to obtain the threshold crossing time from the output response. In the next section we present the sensitivity analysis of the output responses and threshold crossing times for typical on-chips interconnects to interconnect parameters. We conclude in the last section.

OUTPUT RESPONSE CALCULATION

The system (2-4) after scaling (5) can be rewritten as:

$$\frac{\partial \tilde{v}}{\partial y} = \varepsilon i + \frac{\partial i}{\partial \tau}, \qquad \frac{\partial i}{\partial y} = \frac{\partial \tilde{v}}{\partial \tau} \tag{6}$$

$$\tilde{e}(\tau) - \beta i(0,\tau) = -\tilde{v}(0,\tau), \qquad -i(1,\tau) = \frac{C_0}{C_t}\frac{\partial \tilde{v}(1,\tau)}{\partial \tau} \tag{7}$$

The multiple scales method [4] requires expansion of the solution for differential equations into a power series of perturbation parameter, which values are relatively small. Additionally, new space variables are introduced. In our analysis we assume the line resistance in low resistance interconnects is small compared to the lossless line impedance Z_0 (Z_0>R_t), the perturbation parameter is ε, and we limit the expansion to two terms and two space variables. For the system of equations (2) we have

$$\tilde{v}(y,\tau) = \tilde{v}_0(y_0,y_1,\tau) + \varepsilon \cdot \tilde{v}_1(y_0,y_1,\tau),$$
$$i(y,\tau) = i_0(y_0,y_1,\tau) + \varepsilon \cdot i_1(y_0,y_1,\tau),$$
$$y_0 = y, \quad y_1 = \varepsilon \cdot y.$$

Than we obtain new systems of differential equation, the result of alternation equation depend on previous equation. The method allows to compute the lossless line output step response comply the losses parameter ε. The way of calculating the step output response for such a system is presented in [3]. For the first travelling wave we obtain:

$$v_{out}(d,t) = v_{01}(d,t) + v_{11}(d,t) =$$
$$= \frac{E_0}{\beta+1}\left(A\cdot\left(\frac{t-T}{T}\right)\cdot e^{-\frac{\alpha}{T}(t-T)} - B\cdot\left(1-e^{-\frac{\alpha}{T}(t-T)}\right)\right)\cdot \mathbf{1}(t-T) \tag{8}$$

for time 0<t<3T

where:

$$A = -\varepsilon\cdot\alpha\cdot e^{-1.5\varepsilon}, \quad B = \frac{\varepsilon}{2}e^{-0.5\varepsilon}\left(1+e^{-\varepsilon}\right) - \frac{\beta\varepsilon}{\beta+1} - 2e^{-0.5\varepsilon}$$

The simulations show that the approximation gives good results, especially during the rise time. When the aim is to calculate the threshold crossing time or to predict the behaviour of the interconnect in that range of time, the accuracy of formula (8) is enough. An example of step response for typical interconnect (parameters taken from [4]) is presented on fig 1.

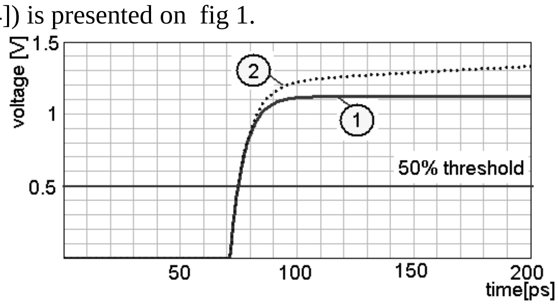

Fig. 1. Step response for the interconnect (R_t=25Ω, L_t=5nH, C_t=1pF, C_0=0.1pF, R_w=25Ω) the output response calculated with (8)- curve 1 compared with SPICE simulation- curve 2.

The fast circuits generates the rising time of the input comparable to the time delay, then in many on-chip interconnects we have to take into account the influence of the input voltage rise time Tr. We model such a problem with the ramp excitation:

$$v_{in}(t) = \frac{E_0}{T_r}t\cdot 1(t) - \frac{E_0}{T_r}\left(t-T_r\right)\cdot 1(t-T_r) \tag{9}$$

We must consider two cases – one for 0<t<T_r, and the other for t>T_r. Denoting the response to ramp excitation at the end of the interconnect as $v^r_{out}(t,d)$ and we have the final output voltage signal [3]:

$$v^r_{out}(t) = \frac{E_0}{(\beta+1)T_r}\left[k\left(1-e^{-\alpha\frac{t-T}{T}}\right) - \frac{t-T}{T}\left(B+\frac{A}{\alpha}e^{-\alpha\frac{t-T}{T}}\right)\right],$$
$$for \quad t-T \le T_r$$

$$v^r_{out}(t) = \frac{E_0}{(\beta+1)T_r}\left[\left(\left(\frac{t-T-T_r}{T\alpha}\right)A + k\right)e^{-\alpha\left(\frac{t-T_r-T}{T}\right)} - \left(\frac{t-T}{T\alpha}A+k\right)e^{-\alpha\frac{t-T}{T}} - \frac{T_r}{T}B\right],$$
$$for \quad t-T \ge T_r \tag{10}$$

where $k = \left(\frac{A}{\alpha^2}+\frac{B}{\alpha}\right)$.

The presented approach allows calculating the approximated ramp excitation response. During the simulation we noticed that some improvement could be obtained if the perturbation parameter is scaled to $\varepsilon' = 3/5\cdot\varepsilon$ [3]. The exemplary ramp response with ε improvement is presented on fig 2.

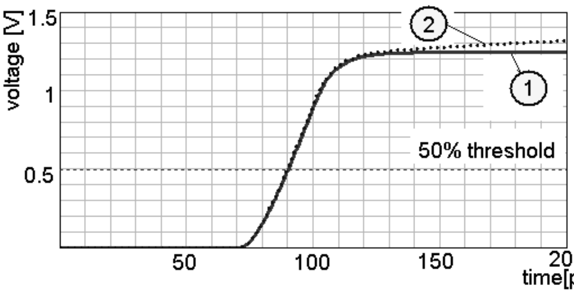

Fig. 2. Ramp response for the interconnect (R_t=25Ω, L_t=5nH, C_t=1pF, C_0=0.1pF, R_w=25Ω, T_r=30ps) the output response calculated with (8)- curve 1 compared with SPICE simulation- curve 2.

THRESHOLD CROSSING TIME CALCULATION

The computation of threshold crossing time is a challenge many authors try to deal with e.g. [2,4]. For the ramp response (10) we can calculate the threshold crossing time, during the ramp times (t<Tr) solving (11) with respect to t:

$$\rho = \frac{E_0 k}{(\beta+1)T_r}\left(1-e^{-\alpha\frac{t-T}{T}}\right)-\frac{t-T}{T}\left(B+\frac{A}{\alpha}e^{-\alpha\frac{t-T}{T}}\right), \qquad (11)$$

where ρ is the threshold value (0÷1).
Equation (11) can be reduced to the form

$$z \cdot e^{z} = a, \quad z = W(a) \qquad (12)$$

where a is a constant, and z is the time function.
The left side of (12) is the function of z only and does not depend on the interconnect parameters. To solve the equation (12) and obtain the threshold crossing time we approximate W(a) Lambert function, occurring in solution of (12) [3]. The approximation for the step and ramp response must be done separately. The threshold crossing times for typical interconnect parameters taken from [4] give very good accuracy for the perturbation parameter ε < 1. For the ramp response the threshold crossing time is given by:

$$\tau_\rho = \left(1+\frac{K_q - W(x)}{\alpha}\right)\cdot T \qquad (13)$$

Where the x and K_q are constant dependent on interconnect parameters and the W(x) function is approximated by:

$$W(x) = \frac{1}{u_1}\ln\left(\frac{x}{u_0}\right)$$

where u_1=-2.988, u_0=-0.887.

SENSITIVITY ANALYSIS

Simulations using in investigation of the interconnect response, both the output response and threshold crossing time calculation can be highly sensitive to small changes in the parameter values. Due to generation by the presented approach the closed form formulas for output response (8) and (10), we could calculate the sensitivity of the output response with the formula:

$$S_v^{\lambda} = \frac{\lambda}{v}\frac{\partial v}{\partial \lambda} \qquad (14a)$$

and for the threshold crossing time:

$$S_{t\rho}^{\lambda} = \frac{x}{t_\rho}\frac{\partial t_\rho}{\partial x} \qquad (14b)$$

In our work we present the sensitivity of the successive functions to ε, β and α respectively. The parameters represents the losses in the interconnect R, the input gate resistance R_w and the output gate capacitance C_0 respectively.

Sensitivity analysis of the step response

The step response (8) calculated for the typical low loss interconnect parameters taken from [4] The transmission line model parameters: R_t=25Ω, L_t=5nH, C_t=1pF, C_0=0.1pF, R_w=25Ω, $\beta = 1$, $\varepsilon = 0.35$, $\alpha = 10$. The sensitivity of parameters do not exceed the 1 value, and is the highest in the t=T. Some exemplary sensitivity values are presented below (Fig 3, 4 and 5).

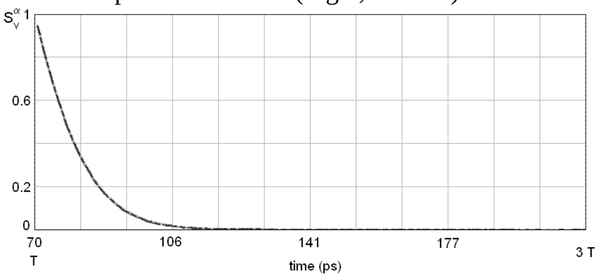

Fig. 3. Sensitivity of the step response to α in time function, for different β values ($\beta, 2\beta, 3\beta, 0.5\beta$)

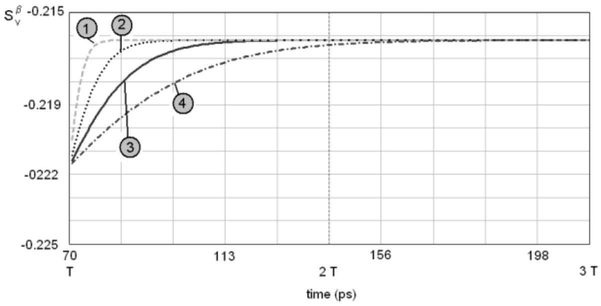

Fig. 4. Sensitivity of the step response to β in function of t for different α values (curves: $1-\alpha$, $2-2\alpha$, $3-5\alpha$, $4-0.5\alpha$), $t = 1.5T$

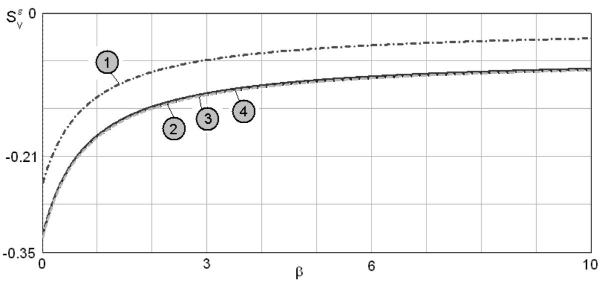

Fig. 5. Sensitivity of the step response to ε in function of β for different α values (curves: $1-0.1\alpha$, $2-0.5\alpha$, $3-\alpha$, $4-5\alpha$), $t = 1.5T$

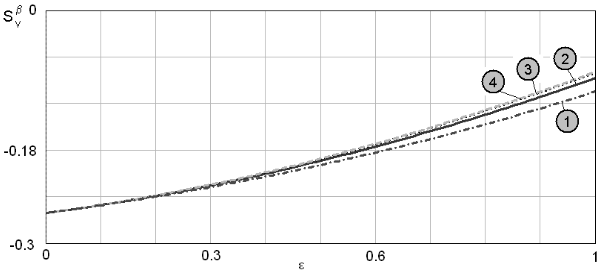

*Fig. 6. Sensitivity of the step response to β **in function of ε for different α values (curves:*** $1-0.5\alpha$, $2-\alpha$, $3-2\alpha$, $4-3\alpha$), $t = 1.5T$.

Sensitivity analysis of the ramp response

The ramp response (10) calculated for the typical low loss interconnect parameters presented in the previous examples. The input rise time is given as τ_r =30ps.

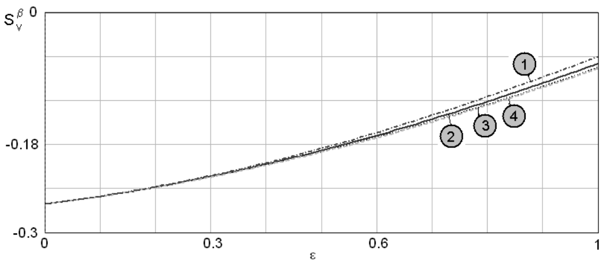

Fig. 7. Sensitivity of the ramp response to β ***in function of*** ε ***for different*** α ***values (curves:*** $1-0.5\alpha, 2-\alpha, 3-2\alpha, 4-3\alpha$***),*** $t=1.5T$ **.**

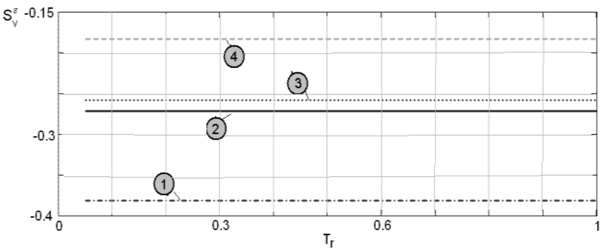

Fig. 8. Sensitivity of the step response to ε ***in function of T.r for different*** α ***values***
(curves: $1-0.1\alpha, 2-\alpha, 3-2\alpha, 4-3\alpha$ ***),*** $t=1.5T$ **.**

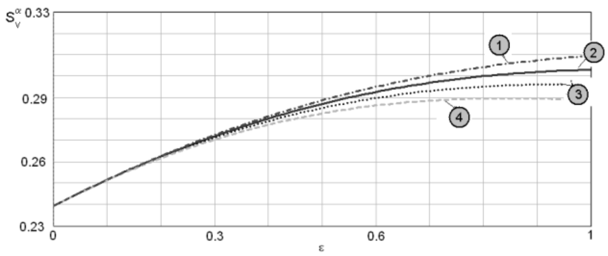

Fig. 9. Sensitivity of the ramp response to α ***in function of*** ε ***for different*** β ***values (curves:*** $1-0.5\beta, 2-\beta, 3-2\beta, 4-3\beta$***),*** $t=1.5T$ **.**

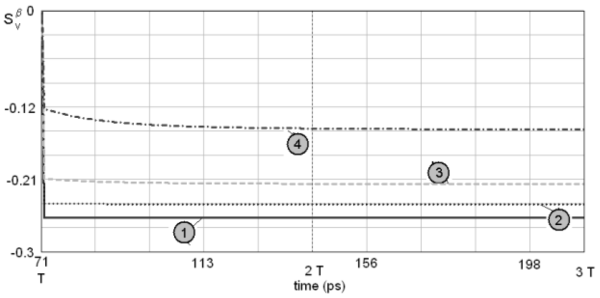

Fig. 10. Sensitivity of the ramp response to β ***in function of time for different*** ε ***values (curves:*** $1-0.1\varepsilon, 2-0.5\varepsilon, 3-\varepsilon, 4-2\varepsilon$ ***)***

Sensitivity analysis of the threshold crossing time for the ramp response

Below we present the sensitivity analysis for the threshold time formula. To show that the approximation of W(x) function can be use in threshold crossing time calculation, we present the sensitivity of the threshold time to the u_0 and u_1 constants in α for different ε values (Fig 11 and 12). The similar results we obtain in beta functions.

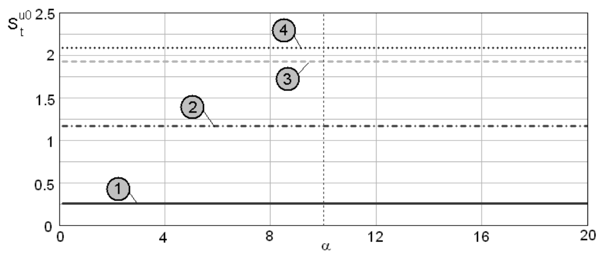

Fig. 11. Sensitivity of the threshold crossing time to u_0 *in function of* α ***for different*** ε ***values (curves:*** $1-0.1\varepsilon, 2-0.5\varepsilon, 3-\varepsilon, 4-2\varepsilon$ ***),*** $t=1.5T$ **.**

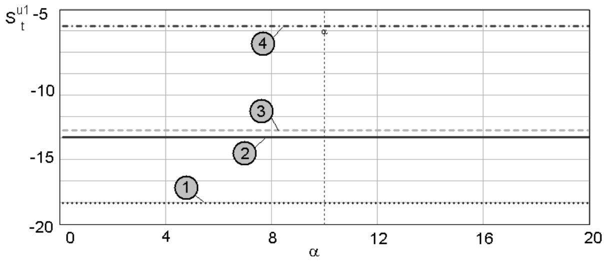

Fig. 12. Sensitivity of the threshold crossing time to u_1 *in function of* α ***for different*** ε ***values (curves:*** $1-0.1\varepsilon, 2-0.5\varepsilon, 3-\varepsilon, 4-2\varepsilon$ ***),*** $t=1.5T$ **.**

The most sensitive is the threshold crossing time to the approximation coefficients in threshold function and epsilon. To ensure the good approximation the threshold time function for different ρ values separately the approximation should be done. The last simulations (Fig 11) shows the sensitivity of threshold crossing time to β and ε for various parameter functions. We could observe, that the sensitivity for rise time is almost zero for wide range of parameters (exemplary function of sensitivity for various α is presented in fig 15).

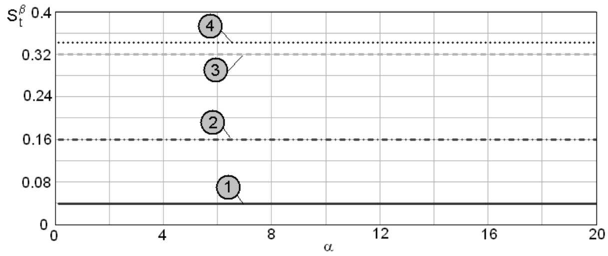

Fig. 13. Sensitivity of the threshold crossing time to β *in* α *function, for different* ε ***values (curves:*** $1-0.1\varepsilon, 2-0.5\varepsilon, 3-\varepsilon, 4-2\varepsilon$ ***),*** $t=1.5T$.

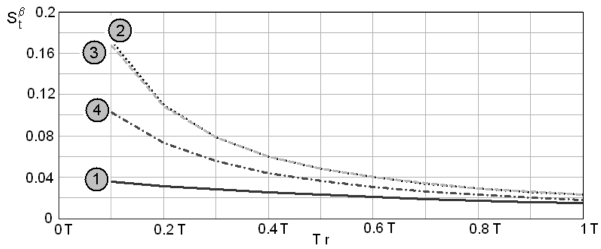

Fig. 14. Sensitivity of the threshold crossing time to β *in* τ_r *function, for different* ε ***values (curves:*** $1-0.1\varepsilon, 2-0.5\varepsilon, 3-\varepsilon, 4-2\varepsilon$ ***),*** $t=1.5T$.

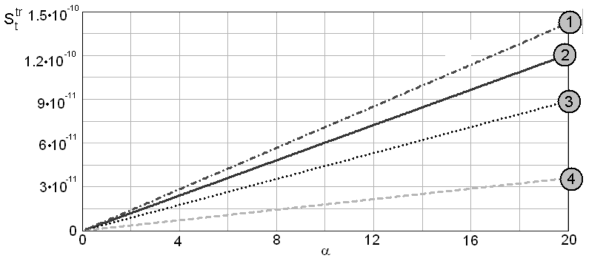

Fig. 15. Sensitivity of the threshold crossing time to ***T.r*** *in* α *function, for different* β *values* ***(curves:*** $1-0.5\beta,\ 2-\beta,\ 3-2\beta,\ 4-5\beta$***)*** .

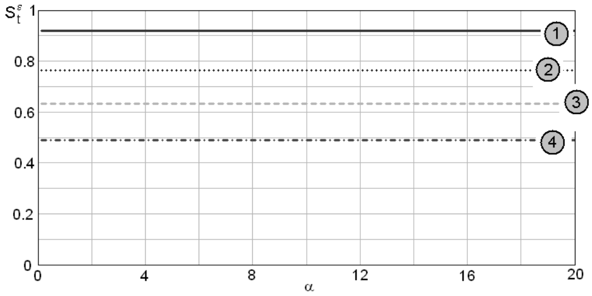

Fig. 16. Sensitivity of the threshold crossing time to ε *in* α *function, for different* β *values* ***(curves:*** $1-0.1\beta, 2-0.5\beta, 3-\beta, 4-2\beta$***).***

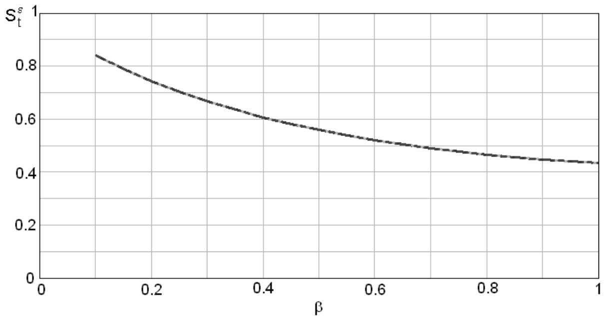

Fig. 17. Sensitivity of the threshold crossing time to ε *in* β *function, for different* α *values (* $0.1\alpha,\ 0.5\alpha,\ \alpha,\ 2\alpha$ *)* **.**

CONCLUSIONS

In the paper we present the sensitivity analysis for the step and ramp response of the interconnect, and for the threshold time. The analysis show, that the presented approach is not very sensitive to the interconnect parameter changes in the range for the typical lowloss parameters ($\varepsilon < 1$). The sensitivity of threshold time formula is valid for $\beta < 1$ what is still accurate for modern clock interconnects. We have also shown, that the sensitivity of the threshold crossing time formula does not change

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REFERENCES

[1] Deutsch A. et al., “Effects of noise on timing or data-pattern dependent delay variation when transmission-line effects are taken into account for on-chip wiring”, Proceedings of the 11th of IEEE Workshop on Signal Propagation On Interconnects, May 2007, Genova, pp. 7-11.

[2] Ligocka, A. Bandurski, W. “Estimation of threshold crossing time in low-loss interconnects”, Proceedings of the Mixed Design of Integrated Circuits and System, 2006. MIXDES 2006. Proceedings of the International Conference 22-24 June 2006, pp 469- 474

[3] Ligocka A. Bandurski, W. Rydlichowski P. „A New Approach to Analysis and Simulation of Single and Coupled Low-Loss Interconnects.” Proceedings of ECCTD 2007, Sevilla, August 26 - 30, 2007

[4] Ismail Y.I., Friedman E.G., “Effects of inductance on the propagation delay and repeater insertion in VLSI circuits”, IEEE Transactions on VLSI Systems, vol. 8, No. 2, April 2000, pp. 195-206

[5] Ismail Y. I., Amin C., "Computation of signal threshold crossing times directly from higher order moments", IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems (TCAD), vol. 23, no. 8, pp. 1264-1276, August 2004.

[6] Shivamoggi B.K. “Perturbation methods for differential equations”, Birkhauser, Boston 2003.