Multiconductor Transmission Line Scattering Parameters in Time Domain

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Abstract— In the paper, we present a fast and effective calculation method of coupled interconnect S-parameters. The paper presents an approach based on the method of successive approximations, but taking into account the dependence on the frequency of line parameters. The concept is to use the rational approximation of the matrix of per-unit-length parameter of the line calculated for each frequency. In our approach, we calculate the scattering parameters of an n-wire transmission line.

Keywords — Interconnect, VLSI, Scattering Parameters, transmission line

III. INTRODUCTION

The modeling of transmission lines in the time-domain is an ongoing challenge for people involved in the simulation of integrated circuits and/or printed circuit boards at a high frequency. The literature on this subject is very vast and presenting it here is almost impossible. Among many methods and approaches, we would like to focus on two that include further references. In the first paper [1], the author presents an approach based on the dyadic Green's function and vector fitting of per-unit-length impedances and admittances of transmission line to obtain the Z matrix of an n-port of the multiconductor transmission line. Every entry of the Z matrix is the sum of rational functions of the complex frequency s, which facilitates the transformation to the time-domain and circuit modeling in SPICE. The biggest problem is the necessity to take into account a large number of terms in every entry of the mentioned Z matrix. On the other hand, paper [2] developed a method of conversion of differential telegrapher's equations into integral equations and next solving them by through successive approximation. In that approach, we obtain first order approximation of the solution in a simple analytical form, which is valid for low-loss transmission lines. The drawback of that approach was not including the skin effect and dielectric dispersion.

In this paper, we present an improved version of the approach based on the method of successive approximations [2], taking into account the line parameter dependence on frequency. For this purpose, as in [1], we use the concept of rational approximation of the matrix of the per-unit-length parameter of the line calculated for each frequency. Our approach is based on scattering parameters of an n-wire transmission line.

The paper is organized as follows. The next section presents the integral equations approach to the dispersive transmission line. In the third section, we employ the method of successive approximation to calculate the scattering parameters of a multiconductor line. In the fourth section, some examples are given. We make conclusions in the last section.

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IV. TELEGRAPHER'S EQUATIONS IN INTEGRAL FORM

A. *Telegrapher's equations for the dispersive multiconductor transmission line*

Let us consider a multiconductor transmission line, consisting of N+1 conductors of which one is considered as a reference one. The telegrapher's equations are as follows:

$$-\frac{d\mathbf{V}(\mathrm{p,y})}{dy} = \left(\boldsymbol{Z_o}(\mathrm{p}) + \boldsymbol{Z_1}(\mathrm{p})\right)\boldsymbol{I}(\mathrm{p,z})$$
$$-\frac{d\boldsymbol{I}(\mathrm{p,y})}{dy} = \left(\boldsymbol{Y_o}(\mathrm{p}) + \boldsymbol{Y_1}(\mathrm{p})\right)\boldsymbol{V}(\mathrm{p,y}) \tag{1}$$

where

$$\boldsymbol{Z}_o(\mathrm{p}) = \boldsymbol{R} + p\boldsymbol{L}, \qquad \boldsymbol{Y_o}(\mathrm{p}) = \boldsymbol{G} + p\boldsymbol{C}$$

$$\boldsymbol{Z}_1(\mathrm{p}) = \sum_{m=1}^{N_z}\frac{\boldsymbol{R}_m^z}{p + p_m^z}, \boldsymbol{Y_1}(\mathrm{p}) = \sum_{m=1}^{N_y}\frac{\boldsymbol{R}_m^y}{p + p_m^y}$$

$y = z/d$, $\tau = {}^{t}/_{T}$, $p = sT, T = d\sqrt{L_{11}^o C_{11}^o}$

d - length of the line

L_{11}^o - entry of original inductance matrix

C_{11}^o - entry of original capacitance matrix

In (1), Matrices $\mathbf{Z}_1$ and $\mathbf{Y}_1$ are a rational form of per-unit-length impedance and admittance of the multiconductor transmission line obtained as in [1] by means of the vector fitting technique [4]. The next step is partial decoupling of the multiconductor transmission line. It is done, as e.g. in [2], by way of matrix transformations:

$$U = XV, \qquad J = P^{-1} I$$

$$-\frac{dU(\mathrm{p,y})}{dy} = (X^{-1}RP^{-1} + pX^{-1}LP^{-1} + X^{-1}Z_1(\mathrm{p})P^{-1})J(\mathrm{p,z}), \qquad (2)$$

$$-\frac{dJ(\mathrm{p,y})}{dy} = (PGX + pPCX + PY_1(\mathrm{p})X)V(\mathrm{p,y}),$$

where:

$$X = L^{\frac{1}{2}} W diag\left[\frac{1}{\sqrt{\lambda_k}}\right],$$

$$P = diag\left[1/\sqrt{\lambda_k}\right] W^{-1} L^{1/2},$$

W, λ_k - eigenvector and eigenvalues of matrix $L^{\frac{1}{2}} C L^{\frac{1}{2}}$.

Equation (2) is partially decoupled, matrices $X^{-1}LP^{-1} = PCX$ are diagonal. Now, we introduce current waves through matrix transformation:

$$\begin{bmatrix} I_- \\ I_+ \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \mathbf{1} & -\mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}\begin{bmatrix} U \\ J \end{bmatrix} = S\begin{bmatrix} U \\ J \end{bmatrix}.$$

After some matrix manipulation, we obtain a new form of the telegrapher's equations:

$$\frac{d}{dy}\begin{bmatrix} I_- \\ I_+ \end{bmatrix} + p\begin{bmatrix} -\mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda} \end{bmatrix}\begin{bmatrix} I_- \\ I_+ \end{bmatrix} = \begin{bmatrix} \mathbf{P_1} & -\mathbf{P_2} \\ \mathbf{P_2} & -\mathbf{P_1} \end{bmatrix}\begin{bmatrix} I_- \\ I_+ \end{bmatrix} + \begin{bmatrix} \mathbf{Q_1} & -\mathbf{Q_2} \\ \mathbf{Q_2} & -\mathbf{Q_1} \end{bmatrix}\begin{bmatrix} I_- \\ I_+ \end{bmatrix}, \qquad (3)$$

where

$$\mathbf{P_1} = \frac{1}{2} X^{-1}(X^{-1}RP^{-1} + PGX),$$

$$\mathbf{P_2} = \frac{1}{2} X^{-1}(X^{-1}RP^{-1} - PGX),$$

$$\mathbf{Q_1} = \frac{1}{2} X^{-1}(X^{-1}X^{-1}Z_1(\mathrm{p})P^{-1} + PY_1(\mathrm{p})X),$$

$$\mathbf{Q_2} = \frac{1}{2} X^{-1}(X^{-1}X^{-1}Z_1(\mathrm{p})P^{-1} - PY_1(\mathrm{p})X).$$

In (3), we move the diagonal entries of matrix $\mathbf{P_1}$ to the left side and after some manipulations, we obtain (4).

$$\frac{d}{dy}\left[exp\left[-(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda})\right] I_-\right] = exp\left[-(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda})\right]\mathbf{P'_{10}} I_- - exp\left[-(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda})\right]\mathbf{P'_2} I_+,$$

$$\frac{d}{dy}\left[exp\left[(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda})\right] I_+\right] = exp\left[(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda})\right]\mathbf{P'_2} I_- - exp\left[(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda})\right]\mathbf{P'_{10}} I_+, \qquad (4)$$

where

$$\mathbf{P'_{10}} = \mathbf{P_{10}} + \mathbf{Q_1}, \qquad \mathbf{P'_2} = \mathbf{P_2} + \mathbf{Q_2}.$$

B. Integral equations for the dispersive multiconductor transmission line

By integrating the first equation (4) from y to1 and the second from 0 to y, we obtain:

$$I_-(p,y) = exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda}\right)(1-y)\right] I_-(p,1) - \int_y^1 exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda}\right)(\xi - y)\right]\mathbf{P'_{10}} I_-(p,\xi)d\xi + \int_y^1 exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda}\right)(\xi - y)\right]\mathbf{P'_2} I_+(p,\xi)d\xi, \qquad (5a)$$

$$I_+(p,y) = exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda}\right)y\right] I_+(p,0) + \int_0^y exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda}\right)(y - \xi)\right]\mathbf{P'_{10}} I_-(p,\boldsymbol{\xi})d\boldsymbol{\xi} - \int_0^y exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\mathbf{\Lambda}\right)(y - \xi)\right]\mathbf{P'_2} I_+(p,\boldsymbol{\xi})d\boldsymbol{\xi}\mathrm{q} \qquad (5a)$$

Equations (5) are integral telegrapher's eqs. and can be solved analytically or numerically. We now calculate (5) using the method of successive approximations.

V. SCATTERING PARAMETERS OF THE MULTICONDUCTOR TRANSMISSION LINE

The first order approximation is not difficult to obtain (see[2]). Here, we make a first order approximation of the first of equations (5) and it has the following form:

$$I_-(p,y) = e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(1-y)} I_-(p,1) - \int_y^1 e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(\xi-y)}\,\mathbf{P'_{10}} e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(1-\xi)} d\xi\, I_-(p,1) + \int_y^1 e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(\xi-y)}\,\mathbf{P'_2}\, e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)\xi} d\xi\, I_+(p,0), \qquad (6)$$

Substituting y=0 in (6), we obtain the relationships:

$$I_-(p,0) = \int_0^1 e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(\xi-0)}\,\mathbf{P'_2}\, e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)\xi} d\xi\, I_+(p,0) + \left[e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(1-y)} - \int_0^1 e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(\xi-0)}\,\mathbf{P'_{10}} e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(1-\xi)} d\xi\right] I_-(p,1). \qquad (7)$$

In (7), we can easily identify scattering parameters as:

$$S_1(p) = \int_0^1 e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(\xi-0)}\,\mathbf{P'_2}\, e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)\xi} d\xi$$

$$S_2(p) = e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(1-y)} - \int_0^1 e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)\xi}\,\mathbf{P'_{10}} e^{-\left(\mathbf{P_{1k,k}}+p\Lambda\right)(1-\xi)} d\xi. \qquad (8)$$

A. Scattering matrices in the frequency domain

The calculation of integrals (8) is straightforward and the results are the following

$$S_1(p)_{m,i,j} = \left[P_{2_{i,j}} + Q_2(p)_{m,i,j}\right] \cdot \frac{1 - \exp\left(-\left(P_{1_{i,i}} + P_{1_{j,j}} + p(\lambda_i + \lambda_j)\right)\right)}{P_{1_{i,i}} + P_{1_{j,j}} + p(\lambda_i + \lambda_j)}, \qquad (9a)$$

$$S_2(p)_{m,i,j} = e^{-\left(P_{1_{i,i}} + p\lambda_i\right)} + \left(P_{10_{i,j}} + Q_1(p)_{m,i,j}\right)\frac{e^{-\left(P_{1_{i,i}} + p\lambda_i\right)} + e^{-\left(P_{1_{j,j}} + p\lambda_j\right)}}{P_{1_{i,i}} - P_{1_{j,j}} + p\left(\lambda_i - \lambda_j\right)}, \qquad (9b)$$

where

$Q_{1/2}(p)_{m,i,j} = \frac{1}{2}\left[\frac{Z_{2m,i,j}}{p+p_m^z} +/- \frac{Y_{2m,i,j}}{p+p_m^z}\right],$

$Z_{2m,i,j} = [\boldsymbol{X}^{-1}\boldsymbol{R}_m^z\boldsymbol{P}^{-1}]_{i,j},\ Y_{2m,i,j} = [\boldsymbol{P}\boldsymbol{R}_m^z\boldsymbol{X}]_{i,j},$

B. Scattering matrices in the time domain

Scattering matrices can be transformed to the time domain. Then the scattering matrix $\mathbf{s}_1(\tau)$:

$$\boldsymbol{s}_1(\tau)_{m,i,j} = sA_1(\tau)_{,i,j} + sB_1(\tau)_{m,i,j} \quad (10a)$$

where

$$sA_1(\tau)_{i,j} = \frac{P_{2_{i,j}}}{\lambda_i + \lambda_j} e^{-\frac{a_{i,j}}{Ta_{i,j}}\tau}\left[\mathbf{1}(\tau) - \mathbf{1}(\tau - \lambda_i - \lambda_j)\tau\right],$$

$$sB_1(\tau)_{m,i,j=} = \frac{1}{2(\lambda_i + \lambda_j)}\begin{bmatrix} Z_{2m,i,j}\,(h_{1z}(m,i,j,\tau) \\ -e^{-\frac{a_{i,j}}{Ta_{i,j}}} h_{1z}(m,i,j,\tau - Ta_{i,j})) - \\ Y_{2m,i,j}\,(h_{1y}(m,i,j,\tau) \\ -e^{-\frac{a_{i,j}}{Ta_{i,j}}} h_{1y}(m,i,j,\tau - Ta_{i,j})) \end{bmatrix},$$

and

$$h_{1z}(m,i,j,\tau) = \frac{\mathbf{1}(\tau)e^{-\frac{a_{i,j}}{Ta_{i,j}}\tau} - \mathbf{1}(\tau)e^{-p_m^z\tau}}{p_m^z - \frac{a_{i,j}}{Ta_{i,j}}},$$

$$h_{1y}(m,i,j,\tau) = \frac{\mathbf{1}(\tau)e^{-\frac{b_{i,j}}{Tb_{i,j}}\tau} - \mathbf{1}(\tau)e^{-p_m^y\tau}}{p_m^z - \frac{b_{i,j}}{Tb_{i,j}}},$$

$$a_{i,j} = P_{1_{i,i}} + P_{1_{j,j}}, Ta_{i,j} = \lambda_i + \lambda_j.$$

Scattering matrix $\mathbf{s}_2(\tau)$ takes the form:

$$\boldsymbol{s}_2(\tau)_{m,i,j} = sA_2(\tau)_{i,j} + sB_2\,(\tau)_{i,j} + sC_2(\tau)_{m,i,j}, \quad (10b)$$

where

$$sA_2(\tau)_{i,j} = diag\left[e^{-P_{1i,i}}\delta(\tau - \lambda_i)\right],$$

$$sB_2(\tau)_{i,j} = \frac{P_{10_{i,j}}}{Tb_{i,j}} e^{-\frac{P_{1jj}\lambda_i - P_{1i,i}\lambda_j}{Tb_{i,j}}} e^{-\frac{b_{i,j}}{Tb_{i,j}}\tau}\left[\mathbf{1}(\tau - \lambda_j) - \mathbf{1}(\tau - \lambda_i)\boldsymbol{\tau}\right],$$

$$sC_2(\tau)_{m,i,j} = \frac{1}{2(\lambda_i - \lambda_j)}\begin{bmatrix} Z_{2m,i,j}\,(e^{-P_{1j,j}} h_{2z}(m,i,j,\tau - \lambda_j) \\ -e^{-P_{1i,i}} h_{2z}(m,i,j,\tau - \lambda_i)) + \\ Y_{2m,i,j}\,(e^{-P_{1j,j}} h_{2y}(m,i,j,\tau - \lambda_j) \\ -e^{-P_{1i,i}} h_{2y}(m,i,j,\tau - \lambda_i)) \end{bmatrix},$$

and

$$h_{2z}(m,i,j,\tau) = \frac{\mathbf{1}(\tau)e^{-\frac{b_{i,j}}{Tb_{i,j}}\tau} - \mathbf{1}(\tau)e^{-p_m^z\tau}}{p_m^z - \frac{b_{i,j}}{Tb_{i,j}}},$$

$$h_{2y}(m,i,j,\tau) = \frac{\mathbf{1}(\tau)e^{-\frac{a_{i,j}}{Ta_{i,j}}\tau} - \mathbf{1}(\tau)e^{-p_m^y\tau}}{p_m^y - \frac{a_{i,j}}{Ta_{i,j}}},$$

$$b_{i,j} = P_{1_{i,i}} - P_{1_{j,j}}, Tb_{i,j} = \lambda_i - \lambda_j.$$

IV. Results

As an example, we have considered a three-wire transmission line (microstrip transmission line) shown in Fig.1.

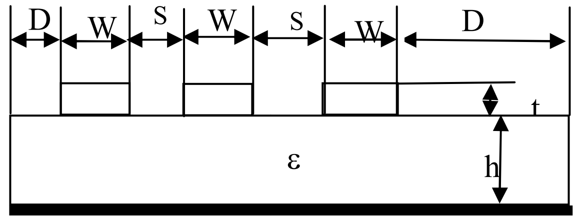

Fig.1 Microstrip data: W_1=100µm, W_2=100µm, W_3=600µm, h=625 µm, s_1=300µm, s_2=280µm, t=30µm tgδ=10^{-4}, ε_r=4.7, D_l= 800 µm, D_2=800 µm, d=0.2m.

The per-unit-length matrices $\mathbf{Z}_o$, $\mathbf{Z}_1$, $\mathbf{Y}_o$ and $\mathbf{Y}_1$ were calculated by means of the LINPAR programme [3] in seventeen frequency points from 10Hz to 2.1GHz. Next, an approximation was performed by rational functions using the vector fitting algorithm [4] to obtain the form as in (1). The approximated matrices $\mathbf{Z}_o$, $\mathbf{Z}_1$, $\mathbf{Y}_o$, and $\mathbf{Y}_1$ were then used to calculate all parameters needed for the calculation of scattering matrices $\mathbf{S}_1$(p) and $\mathbf{S}_2$(p) in the frequency domain and $\mathbf{s}_1(\tau)$ and $\mathbf{s}_2(\tau)$ in the time domain using formulas (9) and (10), respectively. The exemplary results in the frequency domain are shown in Figs. 2, 3. The comparison to the numerical integration of integral equations (5) is shown in the Figs. 4, 5. The exemplary time domain scattering parameters are presented in Figs. 6, 7.

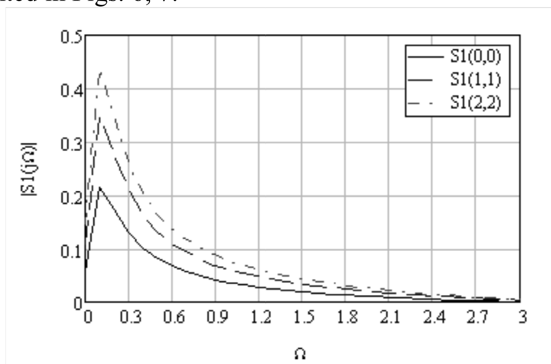

Fig. 2 Diagonal entries of scattering matrix $\mathbf{S}_1$(jΩ_n) in frequency domain, Ω - normalized frequency.

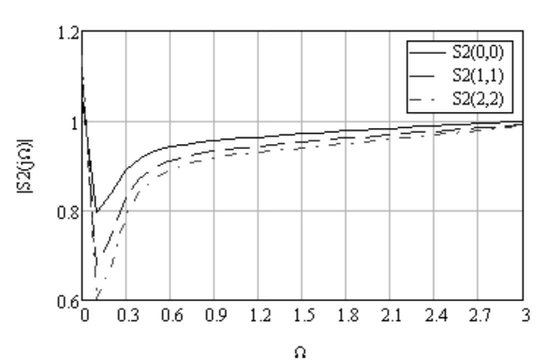

Fig.3 Diagonal entries of scattering matrix $\mathbf{S}_2$(jΩ_n) in frequency domain, Ω - normalized frequency.

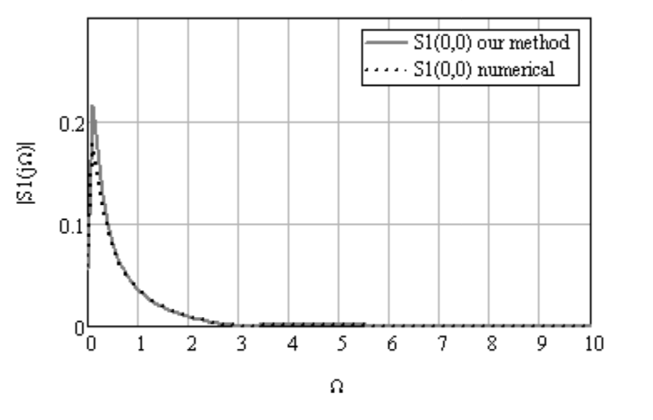

Fig.4 Diagonal entries of scattering matrix $\mathbf{S}_1(j\Omega_n)_{0,0}$ in frequency domain, Ω - normalized frequency.

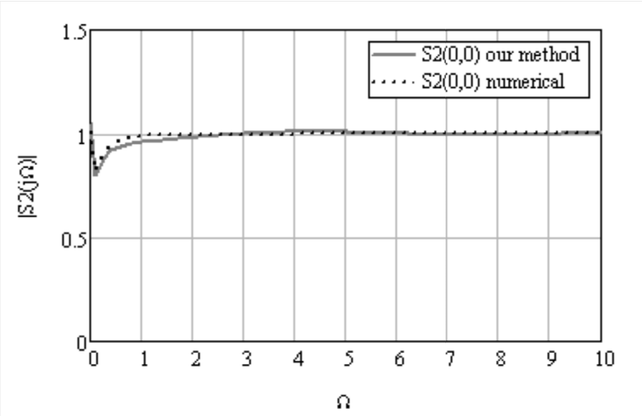

Fig.5 Diagonal entries of scattering matrix $\mathbf{S}_2(j\Omega_n)_{0,0}$ in frequency domain, Ω - normalized frequency.

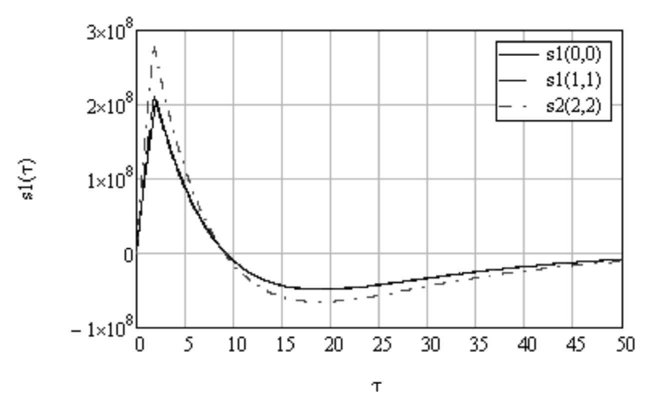

Fig.6 Diagonal entries of scattering matrix $\mathbf{s}_1(\tau)$ in time domain, τ - normalized time.

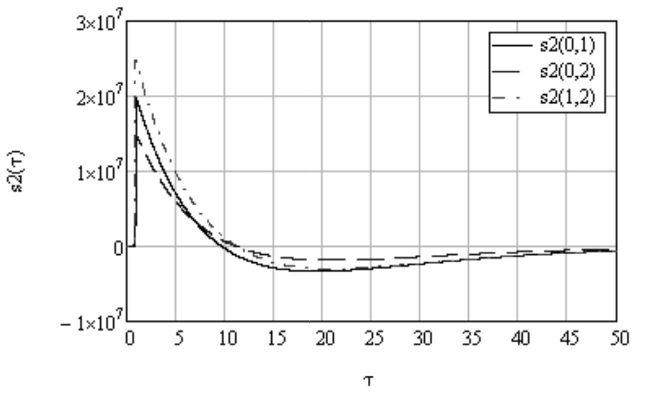

Fig.7 Off-diagonal entries of scattering matrix $\mathbf{s}_2(\tau)$ in time domain τ - normalized time.

V. CONCLUSIONS

We have shown that it is possible to employ a generalized approach based on the method of successive approximation for the case of a multiconductor transmission line with frequency dependent parameters. As a result, we obtain a closed form (it means first order approximation) of scattering parameters of the multiconductor transmission line (first order approximation), both in the frequency and time domains. In the case of low loss, such an approximation is satisfactory. Compared to the approach based on the dyadic green function [1], the presented approach is simpler, of course assuming sufficiently small losses of the multiconductor transmission line. The presented approach permits the implementation of the model in SPICE.

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