Impulse Response of Transition Zone Diffraction on Many Convex Obstacles in Cascade

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Abstract— **The paper presents the derivation of a 2D time-domain model of two bare conducting convex obstacles for the case of UWB baseband signal propagation. The model includes amplitude diffraction and first order transition zone diffraction, called the slope diffraction. The considered model is represented by its impulse response. Thanks to the introduction of some vital approximations of the expressions occurring in amplitude term and slope term of the impulse response, it can be given in a closed form. The presented approach is extended for the case of more than two cascaded convex obstacles in the channel. The Uniform Theory of Diffraction (UTD) formulated in the frequency domain is used in the derivation of the model. The correctness and accuracy of the derived model is verified by simulation of an Ultra Wide Band (UWB) pulse distortion.**

I. INTRODUCTION

The paper presents the time domain (TD) modeling of diffraction caused by convex objects. We consider the soft polarization case. We propose to use deterministic UWB channel modeling for the channel with convex objects. Examples of such objects are round buildings, round pillars in buildings and rounded corridors in buildings. In our considerations, the transition zone diffraction is included. Therefore the slope diffraction is the key factor and cannot be omitted. When the UWB signal propagation is taken into account, the time domain modeling is the right choice.

In the we present the closed form formula for the impulse response of two cascaded, perfectly conducting convex obstacles shadowing the transmitter and the receiver. We base on the impulse response of two convex conducting obstacles derived in [1]. When the latter is applied to a calculation of a response of the convex obstacles for an incident UWB signal, the complexity of a particular simulation of the UWB pulse propagation can be too high for more complex scenarios. When bigger amount of convex obstacles occur in a signal path, the complexity of calculations grows fast. To overcome this problem we present in this paper the improvement of the impulse response derived in [1]. We start with the formula for the impulse response given in [1] and transform it into a lot more profitable form for UWB EM wave propagation tools. For the purpose of doing it we introduce also some crucial approximations whose application enables to simplify the impulse response. After showing how the new formula for two convex obstacles impulse response look like we describe the way of derivation of the impulse responses of more than two convex obstacles.

In Section 2 of the paper there is shown the geometrical model of two convex obstacles for the case of transition zone diffraction, as well as the impulse response of the model presented in [1]. Section 3 gives the method of transforming the impulse response of two convex obstacles into a lot less complex form and extension of the method to more than two convex obstacles. The simulations results are presented in Section 4. Section 5 gives the conclusions.

II. THE MODEL OF TWO CONVEX OBSTACLES FOR TRANSITION ZONE CASE

The model of two cascaded convex obstacles is shown in Fig. 1. The parameters of the convex obstacles are described in [3]. When two convex obstacles are taken into account, and the second obstacle is in the transition zone of the first convex obstacle, the frequency response of the "partial" channel has the following form [1]:

$$[H_s(\omega,s)+H_A(\omega)]A(s)e^{-jk(\theta_1 R_{H1}+s)},$$

(2.1)

where s is equal to distance $|Q_1Q_2'|$, A(s) is the spreading factor, the input of the channel is at point Q_1' and the output

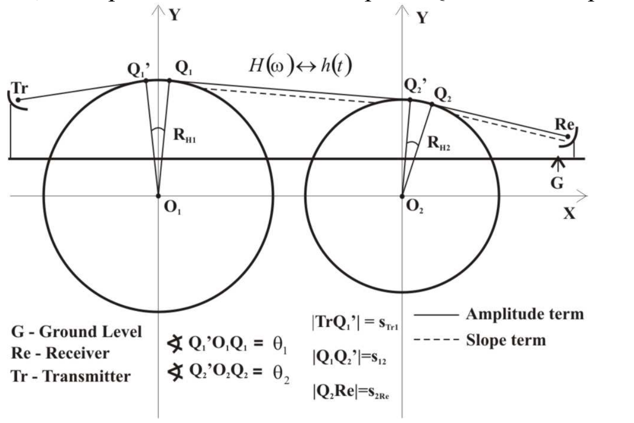

Fig. 1 Two cascaded convex obstacles shadowing the transmitter and receiver.

of the channel is at point Q_2 (Fig. 1). The slope term and amplitude term of (2.1) are related through the following equation:

$$H_s(\omega,s)=\frac{\partial H_A(\theta)}{s\partial\theta} \qquad (2.2)$$

The formulas for $H_A(\omega)$ and $H_S(\omega,s)$ are given in [1]. In the model we assume that the slope of the field is zero when the field originates from the transmitter. We further assume that the receiver is in the far zone, so the slope of the field originating from the second obstacle equals zero. Then the TD field at the receiver is defined by (2.3) [1,2] (time delay of the signal over the distance $\theta_{1/2}R_{1/2}$ is not included):

$$E_{\mathrm{Re}}(t)=\left(E_2(t)*h_A(t,L_{A12\mathrm{Re}},R_2,\theta_2)+\frac{\partial E_2(t)}{\partial n}*d(t,L_{s12\mathrm{Re}},R_2,\theta_2)\right)\ldots \qquad (2.3)$$
$$A_2(s)*\delta\left(t-\frac{s_{2\mathrm{Re}}}{c}\right),$$

where:

$$E_2(t)=E_1(t)*h_A(t,L_{ATr12},R_1,\theta_1)*\delta\left(t-\frac{s_{12}}{c}\right)A_1(s),$$

$$\frac{\partial E_2(t)}{\partial n}=E_1(t)*h_s(t,L_{sTe12},R_1,\theta_1,s_{12})*\delta\left(t-\frac{s_{12}}{c}\right)A_1(s).$$

E_1 is the function of the field at point Q_1', where the creeping ray hits the first convex object. L_{A12Re} and L_{S12Re} are the distance parameters ensuring the continuity of the amplitude term and slope term of the field around the shadow boundary after the second convex object [2], and L_{ATr12} and L_{STr12} are the distance parameters ensuring the continuity of the amplitude term and slope term of the field around the shadow boundary after the first convex object [2].

When the spreading factor and delay terms are omitted the impulse response of the two convex obstacles is given by;

$$h(t)=h_A(t,L_{ATr12},R_1,\theta_1)*h_A(t,L_{A12\mathrm{Re}},R_2,\theta_2)+\ldots \qquad (2.4)$$
$$h_s(t,L_{sTe12},R_1,\theta_1,s_{12})*d(t,L_{s12\mathrm{Re}},R_2,\theta_2)$$

For baseband UWB propagation the formulations for the factors occurring in (2.4) have the following forms:

$$h_A(t,N)=A(L,R,\theta)\cdot\frac{1}{\sqrt{t}(t+a(L,\theta))}+\sum_{n=0}^{N}Aq_n(L,R,\theta)\cdot t^{-\left(\frac{n}{3}+\frac{5}{6}\right)} \qquad (2.5)$$

$$h_S(t,s,N)=S_1(L,R,\theta,s)\cdot\frac{\partial}{\partial t}\left(\frac{1}{\sqrt{t}(t+a(L,\theta))}\right)+S_2(L,R,\theta,s)\cdot\frac{1}{\left(\sqrt{t}\right)^3} \qquad (2.6)$$
$$+\sum_{n=0}^{N}Sq_n(L,R,\theta,s)\cdot t^{-\left(\frac{n}{3}-\frac{5}{6}\right)}$$

$$h_S(t,s,N)=D_1(L,R,\theta,s)\cdot\frac{1}{\sqrt{t}(t+a(L,\theta))}+D_2(L,R,\theta,s)\cdot\frac{1}{\left(\sqrt{t}\right)} \qquad (2.7)$$
$$+\sum_{n=0}^{N}Dq_n(L,R,\theta,s)\cdot t^{-\left(\frac{n}{3}+\frac{1}{6}\right)}$$

where $A(L,R,\theta)$, $Aq_n(L,R,\theta)$, $S_{1/2}(L,R,\theta,s)$, $Sq_n(L,R,\theta,s)$, $D_{1/2}(L,R,\theta,s)$, $Dq_n(L,R,\theta,s)$, are the constants dependent on the convex obstacles parameters and the scenario.

III. Simplifications of the Convex Obstacles Impulse Response

The main contribution to complexity of the calculation of response of two convex obstacles for an UWB pulse give the numerical operations of convolutions occurring in (2.4). In order to solve this problem we calculate these convolutions. A problem arises during calculation of the convolutions of functions containing singularities. If we need to calculate convolutions $x_1(t)*x_2(t)$ and at least one of the functions contain singularity at t=0, we proceed as follows. To remove singularities, we integrate $x_1(t)$ n times (if necessary), and we integrate $x_2(t)$ m times (if necessary). As a result we obtain:

$$\underbrace{\int\ldots\int}_{n} x_1(t)dt*\underbrace{\int\ldots\int}_{m} x_2(t)dt=y_1^{\,n}(t)*y_2^{\,m}(t)=p(t) \qquad (3.1)$$

Next we differentiate p(t) m+n times to obtain:

$$x_1(t)*x_2(t)=\frac{\partial^{n+m}}{t^{n+m}}p(t) \qquad (3.2)$$

The procedure is applied to each convolution appearing in expression (2.4). Most of the functions $y_1^n(t)$ and/or $y_2^m(t)$ are proportional to $t^{1/6}$, $t^{5/6}$, $t^{1/2}$, in other words time cores - ***TCs*** - of most $y_1^n(t)$ and/or $y_2^m(t)$ are $t^{1/6}$, $t^{5/6}$, $t^{1/2}$. Unfortunately, functions $y_1^n(t)$ and/or $y_2^m(t)$ in some cases have TCs equal to $\mathrm{arctg}[(B_o t)^{1/2}]$, where B_i is the constant calculated for i-th convex obstacle in the specific scenario. Then convolution (3.1) cannot be obtained in the analytical way. In that case function $\mathrm{arctg}(t^{1/2})$ is approximated by the following expression:

$$C_a(t)=\begin{cases} a_1^{\,k}B_i t+a_0^{\,k}\sqrt{B_i t} & for \quad B_i t<(B_i t)_T \\ a_3^{\,k}+\dfrac{a_2^{\,k}}{\sqrt{B_i t}} & for \quad B_i t\ge(B_i t)_T \end{cases}, \qquad (3.3)$$

where k=1 or 2.

Parameters a_n^k (n=0, 1, 2, 3) and the threshold argument $(B_i t)_T$ are determined by a minimization algorithm (e.g. genetic algorithm).

After applying approximation (3.3) all $y_1^n(t)$ and $y_2^m(t)$ functions are constituted by the components which can be convoluted analytically. For early times - $B_i t<(B_i t)_T$ – the results of the convolutions of the TCs which have to be calculated in order to derive two convex obstacles impulse response are given in Table I.

Table I
Results of the convolutions of TCs of $y_1^n(t)$ and $y_2^m(t)$ – q(t) for $B_it<(B_it)_T$ for 2 convex obstacles

Convoluted TCs	q(t)	Constants details
$\sqrt{t}*\sqrt{t}$	$\frac{\pi}{8}\cdot t^2$	---
$\sqrt{t}*\sqrt[6]{t}$	$\frac{3}{40}B_1\left(\frac{1}{6},\frac{1}{2}\right)\cdot t^{\frac{5}{3}}$	$B_1\left(\frac{1}{6},\frac{1}{2}\right)=\frac{\Gamma\left(\frac{1}{6}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{2}{3}\right)}$
$\sqrt{t}*\sqrt[6]{t^5}$	$\frac{15}{112}B_1\left(\frac{5}{6},\frac{1}{2}\right)\cdot t^{\frac{7}{3}}$	$B_1\left(\frac{1}{6},\frac{1}{2}\right)=\frac{\Gamma\left(\frac{5}{6}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{4}{3}\right)}$
$\sqrt[6]{t}*\sqrt[6]{t^5}$	$\frac{5}{12}{}_2F_1\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},1\right)\cdot t^2$	${}_2F_1\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},1\right)=\Gamma\left(\frac{5}{6}\right)\Gamma\left(\frac{7}{6}\right)$
$\sqrt[6]{t}*\sqrt[6]{t}$	$\frac{3}{8}{}_2F_1\left(\frac{1}{6},\frac{5}{6},\frac{7}{6},1\right)\cdot t^{\frac{4}{3}}$	${}_2F_1\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},1\right)=\frac{\Gamma\left(\frac{1}{6}\right)\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{1}{3}\right)}$
$\sqrt[6]{t^5}*\sqrt[6]{t^5}$	$\frac{3}{16}{}_2F_1\left(\frac{5}{6},\frac{1}{6},\frac{11}{6},1\right)\cdot t^{\frac{8}{3}}$	${}_2F_1\left(\frac{5}{6},\frac{1}{6},\frac{11}{6},1\right)=\frac{\Gamma\left(\frac{5}{6}\right)\Gamma\left(\frac{11}{6}\right)}{\Gamma\left(\frac{5}{3}\right)}$
$t*t^u$	$\frac{1}{(u+1)(u+2)}\cdot t^{2+u}$	$u\in\left\{0,\frac{1}{6},\frac{1}{2},\frac{5}{6},1\right\}$

The $B_1(a,b)$ and ${}_2F_1(a_1,a_2,b_1,x)$ are the beta function and the generalized hypergeometric function respectively [4].

Using the results from Table I we can give the formulation for the simplified impulse response of two convex obstacles:

$$h_2(t)=h_{2A}(t)+h_{2S}(t) \tag{3.4}$$

where:

$$h_{2A}(t)=A_{21}\cdot t+A_{20}+\sum_{n=0}^{N}Aq_{2n}t^{-\left(\frac{n}{3}-\frac{1}{2}\right)}+\sum_{n=0}^{N}Ap_{2n}t^{-\frac{n/2+n+1}{3}} \tag{3.5}$$

$$h_{2S}(t)=S_{20}+\sum_{n=0}^{N}Sq_{2n}t^{-\left(\frac{n}{3}-\frac{1}{6}\right)}+\sum_{n=0}^{N}Sp_{2n}t^{-\frac{n+1/2+n-1}{3}} \tag{3.6}$$

where / is the operator of integer dividing and A_{21}, A_{20}, Aq_{2n}, Ap_{2n}, S_{20}, Sq_{2n}, Sp_{2n}, are the functions of the parameters of the convex obstacles and the appropriate constants from Table I.

When we have the form of the impulse response of two convex obstacles, we can extend the presented method for more than 2 convex obstacles in cascade. By analysing (3.5) and (3.6) and having the "convolutions form" impulse response for 3 convex obstacles in cascade given by:

$$\begin{aligned}h_3(t)&=\left(h_{2A}(t)+h_{2S}(t)\right)*\left(h_S^2t)*d^3(t)+h_A^3(t)\right)=\\&=h_2(t)*h_S^2t)*d^3(t)+h_2(t)*h_A^3(t)\end{aligned} \tag{3.7}$$

where Superscript 2 or 3 in $h_S^2(t)$, $d^3(t)$ $h_A^3(t)$ means that the functions depend on the parameters related to 2nd or 3rd obstacle, we can give TCs of $y_1^n(t)$ and $y_2^m(t)$ which are new for the case of 3 obstacles in cascade. These are $t^{2/3}$, $t^{1/3}$, t^0 (the rest TCs are the same as for the case of two obstacles in cascade). The difference between the $y_1^n(t)$ and $y_2^m(t)$ related to one convex obstacle and two convex obstacles is in the range of the set of values of m and n. For early times - $B_it<(B_it)_T$ – the results of the convolutions of the TCs which have to be calculated in order to obtain the impulse response of 3 convex obstacles in cascade are given in Table II.

Table II
Results of the convolutions of new TCs of $y_1^n(t)$ and $y_2^m(t)$ – q(t) for $B_it<(B_it)_T$ for 3 convex obstacles

Convoluted TCs	q(t)	Constants details
$\sqrt{t}*\sqrt[3]{t}$	$\frac{6}{55}B_1\left(\frac{1}{3},\frac{1}{2}\right)\cdot t^{\frac{11}{6}}$	$B_1\left(\frac{1}{3},\frac{1}{2}\right)=\frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{5}{6}\right)}$
$\sqrt{t}*\sqrt[3]{t^2}$	$\frac{12}{91}B_1\left(\frac{2}{3},\frac{1}{2}\right)\cdot t^{\frac{13}{6}}$	$B_1\left(\frac{1}{6},\frac{1}{2}\right)=\frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{7}{6}\right)}$
$\sqrt[3]{t}*\sqrt[3]{t^2}$	$\frac{1}{6}{}_2F_1\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},1\right)\cdot t^2$	${}_2F_1\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},1\right)=\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)$
$\sqrt[6]{t}*\sqrt[3]{t}$	$\frac{4}{9}{}_2F_1\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},1\right)\cdot t^{\frac{3}{2}}$	${}_2F_1\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},1\right)=\frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{7}{6}\right)}{\sqrt{\pi}}$
$\sqrt[6]{t^5}*\sqrt[3]{t}$	$\frac{12}{91}{}_2F_1\left(\frac{5}{6},\frac{2}{3},\frac{11}{6},1\right)\cdot t^{\frac{13}{6}}$	${}_2F_1\left(\frac{5}{6},\frac{2}{3},\frac{11}{6},1\right)=\frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{11}{6}\right)}{\Gamma\left(\frac{7}{6}\right)}$
$\sqrt[6]{t}*\sqrt[3]{t^2}$	$\frac{24}{55}{}_2F_1\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},1\right)\cdot t^{\frac{11}{6}}$	${}_2F_1\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},1\right)=\frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{5}{6}\right)}$
$\sqrt[6]{t^5}*\sqrt[3]{t^2}$	$\frac{8}{55}{}_2F_1\left(\frac{5}{6},\frac{1}{3},\frac{11}{6},1\right)\cdot t^{\frac{5}{2}}$	${}_2F_1\left(\frac{5}{6},\frac{1}{3},\frac{11}{6},1\right)=\frac{2\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{11}{6}\right)}{\sqrt{\pi}}$
$t*t^u$	$\frac{1}{(u+1)(u+2)}\cdot t^{2+u}$	$u\in\left\{0,\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{5}{6},1\right\}$
t^0*t^u	$\frac{1}{u+1}\cdot t^{u+1}$	$u\in\left\{0,\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{5}{6},1\right\}$

Analysing Table II and Table I we can give the simplified form of the impulse response for 3 convex obstacles in cascade:

$$h_3(t)=h_{3A}(t)+h_{3S}(t) \tag{3.8}$$

where:

$$h_{3A}(t) = A_{32} \cdot t^2 + \sum_{n=0}^{N} Aq_{3n} t^{-\left(\frac{n}{6}-\frac{3}{2}\right)} \qquad (3.9)$$

$$h_{3S}(t) = \sum_{n=0}^{N} Sq_{2n} t^{-\left(\frac{n}{6}-\frac{7}{3}\right)} \qquad (3.10)$$

where A_{32}, Aq_{3n}, Sq_{3n}, are the functions of the parameters of the convex obstacles and the appropriate constants from Table I and Table II. Following the same procedure the impulse response for M>3 convex obstacles in cascade in "convolution recursive form" is given by (3.11), and in the simplified form by (3.12), (3.13) and (3.14):

$$\begin{aligned} h_M(t) &= \left(h_{M-1A}(t) + h_{M-1S}(t)\right) * \big(h_S^{M-1} t) * d^M(t) + h_A^m(t)\big) = \\ &= h_{M-1}(t) * h_S^{M-1}(t) * d^M(t) + h_{M-1}(t) * h_A^{M-1}(t) \end{aligned} \qquad (3.11)$$

$$h_M(t) = h_{MA}(t) + h_{MS}(t) \qquad (3.12)$$

where:

$$h_{MA}(t) = \sum_{n=0}^{N} Aq_{Mn} t^{-\left(\frac{n+12-8M}{6}\right)} \qquad (3.13)$$

$$h_{MS}(t) = \sum_{n=0}^{N} Sq_{2n} t^{-\left(\frac{n+10-8M}{6}\right)} \qquad (3.14)$$

where Aq_{Mn}, Sq_{M3n}, are the functions of the parameters of the convex obstacles and the appropriate constants from Table I and Table II from and similar to that from Table I and Table II.

IV. VERIFICATION OF THE SIMPLIFIED IMPULSE RESPONSE.

In this section we examine the accuracy of the obtained results. In order to do this, we calculate the distortion of an UWB pulse caused by two cascaded convex obstacles. We set $E_1(t)$ to a particular UWB pulse w(t).

$$w(t) = \left[1 - 4\pi\left(\frac{t-t_c}{a}\right)^2\right] e^{-2\pi\left(\frac{t-t_c}{a}\right)^2} \qquad (4.1)$$

The achieved results are compared with the calculations obtained by means of IFFT (Fig. 3). The distorted UWB pulse is normalized to the incident UWB pulse.

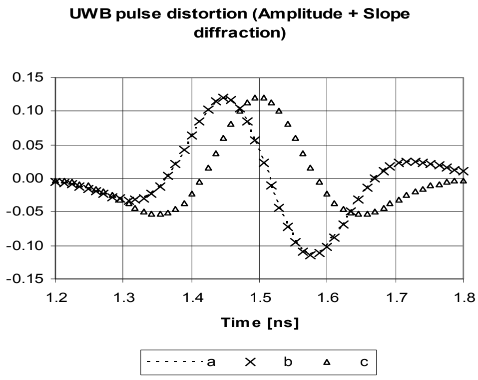

Fig. 3 The shape of the distorted UWB pulse by two convex obstacles with the parameters a=3ns and t_c=1ns. Comparison of the results of the pulse distortion obtained with IFFT (a) with the results of the pulse distortion obtained through direct time domain calculations (b) and with the incident UWB pulse normalized to the amplitude of the distorted pulse (c). The values of parameters of the convex objects are θ_1=0.15, θ_2=0.20, R_1=200m and R_2=150m (Fig. 1).

V. CONCLUSIONS

In the paper we present the procedure for obtaining the simplified time domain formulas for the field at the receiver when two convex obstacles are shadowing the transmitter and the receiver and extension of the procedure for the case of more than 2 convex obstacles in cascade. Numerical experiments show that the results obtained using the approximations presented in Section 3 are accurate compared to the results obtained by (2.4). However the former were obtained at a lot smaller complexity cost and therefore it is a lot more profitable to use it in simulation tools. Although (2.4) relates to the soft polarization case, respective formulas can be derived for hard polarization, using a similar method to that applied for the soft polarization case. Further research should concern the non-good conducting convex obstacles.

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