A New Approach to Polynomial Chaos Expansion for Stochastic Analysis of EM Wave Propagation in an UWB Channel

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***Abstract*—In the paper we present a new universal approach to stochastic simulation of electromagnetic (EM) wave propagation in an ultra-wideband (UWB) channel. We describe and verify our new approach for the case of a diffraction on a convex obstacle, while the approach can applied to any other EM wave propagation phenomenon. We deal with a circular cylinder model of a convex obstacle and uniform theory of diffraction (UTD) which can be effectively used in an asymptotic prediction of EM propagation on convex obstacles. We take advantage of polynomial chaos expansion for statistical analysis. We choose orthonormal basis of Jacobi polynomials as it corresponds to propagation scenario variables that follow Beta stochastic distribution, which is in our opinion the most universal one as it can model Gauss distribution as well as an uniform distribution in a desired variable range.**

***Index Terms*—UWB propagation, Jacobi polynomial chaos, stochastic simulation.**

I. INTRODUCTION

Ultra-wideband technology brought much attention worldwide due to its advantages that can be used in communications and radar area. Nowadays the usual UWB propagation channel concerns indoor scenarios. For the proper analysis of UWB propagation it is important to include the ultra-wideband interaction of EM wave with an obstacle, e.g. convex obstacle. The consequence of this interaction is a signal distortion, which can be neglected in narrow-band EM wave propagation. It is important to have simulation tools that could as accurately as possible predict a field distribution in a given propagation channel so as to enable optimized implementation of an UWB transmission system. The simulators of EM wave propagation base on different models of a propagation environment as empirical, statistical, site-specific, theoretical, etc. In the paper we deal with a theoretical modeling of EM wave indoor propagation. For the presentation of our universal polynomial chaos approach we focus in our analysis on a convex obstacle which can be static, e.g. rounded pillar or non-static, e.g. human. The vital advantage of physical models is enabling of a detailed insight into the influence of wave phenomena and physical parameters of a channel on EM field distribution.

The paper provides a new universal approach to statistical analysis of EM field distribution using polynomial chaos expansion of a given transfer function corresponding to given propagation scenario. We take advantage of the Jacobi polynomial orthonormal basis (in the form of series) as it corresponds to Beta probability density which allows a lot of freedom in description of a stochastic distributions of propagation channel parameters. In particular we can approximate Gauss as well as an uniform probability density with a Beta distribution. In general, expansion coefficients of any transfer function have to be calculated using time consuming numerical integration [1]. Often full wave analysis are required [2, 3]. This analysis need to be performed for each change of probability distribution of propagation scenario parameters. In our approach we obtain a general formula for the expansion coefficients. At the beginning our formula requires calculation of expansion coefficients, by means of integration, for freely chosen Beta distribution of propagation scenario parameters. It allows us then to derive a general analytical formula for the expansion coefficients that can be used for arbitrary Beta distributions of propagation scenario parameters. Consequently we can write that our formula for the expansion coefficients is universal. Although the possible values of a given scenario parameter must be within a predefined limits. For the clarity of description of our universal approach and space saving issue the work presented in this paper focuses on one, selected wave phenomenon. We chose a diffraction phenomenon on a convex obstacle modeled by a circular 2D cylinder. The propagation scenario for this case is presented in Fig. 1.

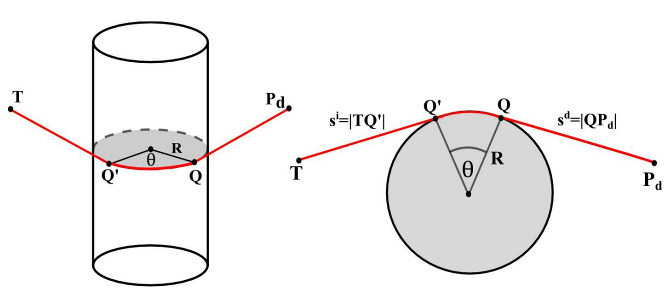

Fig. 1. 2D scenario of a diffraction (creeping) ray traveling along a circular cylinder – a) and cross-section of the cylinder – b).

The presented work has been funded by the Polish Ministry of Science and Higher Education within the status activity task 2016 in "Development of methods for the analysis of propagation and signal processing as well as EMC issues"

In practical applications it is essential to include stochastic properties of physical parameters of propagation channel elements. Different numerical methods enable to include a stochastic behavior of the simulation problem. Among them we can mention Monte Carlo method, moment equations, perturbation techniques, polynomial chaos, etc [1]. The main goal of these techniques is to provide more reliable simulation results dealing with an inaccuracy of a given channel scenario parameters, which are treated as stochastic.

In our analysis we use a polynomial chaos technique, which is very effective and provides simulation results in much less time than the Monte Carlo method. Polynomial chaos expansion has been introduced to computational problems in electromagnetic [2] and recently used in e.g. [3]. To our knowledge no study was focused on application of polynomial chaos for ultra-wideband propagation. Furthermore, in terms of computation complexity, the past results concerning this subject do not provide a very useful in our opinion sort of generality which would express in defining polynomial chaos expansion coefficients by simple analytical formulas which would be functions of parameters of stochastic distributions of propagation scenario variables.

In this paper we present the universal approach to polynomial chaos expansion using orthonormal Jacobi polynomial basis where coefficients of an obtained expansion are functions of parameters of a Beta distribution of a given stochastic variable of a given propagation scenario. We verify the exemplary simulation results obtained with our universal expansion coefficients by comparing it with the results of Monte Carlo Method.

The paper is organized as follows. In Section II we introduce a derivation of an ultra-wideband universal polynomial chaos expansion coefficients using Jacobi polynomials. Section III gives some numerical examples that verify our new coefficients for the case of a diffraction on a convex obstacle modeled by a PEC 2D circular cylinder. We conclude the paper in Section IV.

II. The Universal Polynomial Chaos Expansion Coefficients

Polynomial chaos expansion allows to express a considered function of stochastic variables as a spectral expansion for these variables [1]. When the stochastic variables follow a Beta distribution, with shape parameters α and β, the strong convergence is obtained when orthonormal basis of Jacobi polynomials is used. For stochastic variable $a \le \xi \le b$, with Beta distribution, a Jacobi polynomial expansion of a transfer function $T(\omega_n,\xi)$ for a given pulsation ω_n, can be calculated according to the formula:

$$T(\omega_n,\xi)=\sum_{k=0}^{\infty} a_{k,n} P_k^{\alpha,\beta}\left(f_1(\xi)\right), \tag{1}$$

where:

$$f_1(\xi)=\xi\frac{2}{b-a}-\frac{b+a}{b-a}, \tag{2}$$

while $P_k^{\alpha,\beta}(\xi)$ is a Jacobi polynomial of kth order. It should be noted that Jacobi polynomials are orthogonal in a range of their arguments from -1 to 1, while a weighting function has a Beta distribution shape [1]. The coefficients of an expansion in (1) are calculated by formula:

$$a_{k,n}=\frac{1}{\gamma_k^{\alpha,\beta}}\int_{-1}^{1} T\left(\omega_n, f_2(\xi)\right) P_k^{\alpha,\beta}(\xi)\, w(\alpha,\beta,\xi)\, d\xi\ , \tag{3}$$

where:

$$\gamma_k^{\alpha,\beta}=\int_{-1}^{1} P_k^{\alpha,\beta}(\xi) P_k^{\alpha,\beta}(\xi)\, w(\alpha,\beta,\xi)\, d\xi\ , \tag{4}$$

$$f_2(\xi)=\xi\frac{b-a}{2}+\frac{b+a}{2}, \tag{5}$$

while $w(\alpha,\beta,\xi)$ is a weighting function which describes a Beta distribution:

$$w(\alpha,\beta,\xi)=2^{-(\alpha+\beta+1)}\frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)}(1-\xi)^{\alpha}(1+\xi)^{\beta} \tag{6}$$

Index n in (1) and (3) corresponds to the number of a frequency samples. In our approach arguments a and b used (1) correspond to the range of variable ξ for which we will derive our universal expansion coefficients in subsequent parts of this Section. This range must be wide enough to comprise all possible ξ values which are of our interests. In the scenario of a diffraction ray shown in Fig. 1, we can deal with three stochastic variables, i.e. $\xi=R$, $\xi=\theta$ and $\xi=L_d= s^i s^d (s^i + s^d)^{-1}$. In our numerical examples we will consider $\xi=\theta$ whose range of possible values are $0\le \theta \le \pi$ (actual parameter θ depends on the actual positions of a source and an observation point).

In order to find ultra-wideband expansion coefficients we calculate numerically (3) for all the pulsation samples ω_n in the considered UWB frequency range. In order to approximate a transfer function correctly the number of required expansion coefficients depends on a range of possible values of ξ as well as on a frequency range that are of our interests. The wider are these ranges the more coefficients we need.

The expansion (1) is performed only once for values of ξ with Beta distribution defined on interval $a \le \xi \le b$ and arbitrarily chosen α and β. Then these results are used to obtain our universal expansion coefficients for ξ, with actual Beta distribution, in arbitrary interval, let say $c \le \xi \le d$. The change of an original range of ξ from [a, b] to [c, d] depends on an actual position of a source and an observation point (see Fig. 1).

The goal is to find the following chaotic polynomial expansion:

$$T(\omega_n,\xi)=\sum_{m=0}^{\infty} b_{m,n} P_m^{\alpha,\beta}\left(f_3(\xi)\right), \tag{7}$$

where:

$$f_3(\xi) = \xi \frac{2}{d-c} - \frac{d+c}{d-c}, \tag{8}$$

and

$$b_{m,n} = \frac{1}{\gamma_m^{\alpha,\beta}} \int_{-1}^{1} T(\omega_n, f_4(\xi)) P_m^{\alpha,\beta}(\xi) w(\alpha,\beta,\xi) d\xi , \tag{9}$$

where:

$$f_4(\xi) = \xi \frac{d-c}{2} + \frac{d+c}{2}. \tag{10}$$

It should be noted that parameters α and β will be arguments of (9) and as a consequence their values can be different from those used to calculate (3). For the sake of clarity of the presentation we changed notation from {α, β} in (3) to {α0, β0}. In order to obtain the universal expansion of the transfer function $T(\omega_n,\xi)$ in Jacobi polynomial series, with respect to the random parameter ξ ($c \le \xi \le d$) we eliminate the necessity of recalculation of the integral in equation (9). To do this we perform three steps procedure. First in (9) we make a substitution:

$$b_{m,n} \approx \frac{1}{\gamma_m^{\alpha,\beta}} \int_{-1}^{1} \sum_{k=0}^{K} a_{k,n} P_k^{\alpha 0,\beta 0}(g\xi + h) P_m^{\alpha,\beta}(\xi) w(\alpha,\beta,\xi) d\xi , \tag{11}$$

where:

$$g = \frac{d-c}{b-a} , \tag{12}$$

$$h = \frac{d+c-b-a}{b-a} . \tag{13}$$

It is important to note that infinity upper limit in summation occurring in (1) was substituted by K in (11) as it is enough to have a finite number of expansion coefficients to approximate a given transfer function. In the second step a Jacobi polynomial of kth order is replaced, using a sum of Hermite polynomials of maximum order k [4, 5]. As a result a Jacobi polynomial of order k and parameters α=α0, β=β0 has a following form:

$$P_k^{\alpha 0,\beta 0}(x) = \sum_{s=0}^{k} c_s^k H_s(x), \tag{14a}$$

$$c_s^k = \frac{1}{s!} \int_{-\infty}^{\infty} P_k^{\alpha 0,\beta 0}(x) H_s(x) \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \cdot \tag{14b}$$

Using expansion coefficients (14a) we can rearrange (11) as follows:

$$b_{m,n} = \frac{1}{\gamma_m^{\alpha,\beta}} \int_{-1}^{1} \sum_{k=0}^{K} d_{k,n} H_k(g\xi + h) P_m^{\alpha,\beta}(\xi) w(\alpha,\beta,\xi) d\xi , \tag{15}$$

where $d_{k,n}$ is a combination of (3) and (14b) as follows:

$$d_{k,n} = \sum_{i=k}^{K} a_{i,n} c_k^{\,i} \cdot \tag{16}$$

where k=0,1,2,...,K.
In third step in order to simplify (15) we use the following identity [6]:

$$H_k(x+y) = \sum_{j=0}^{k} \binom{k}{j} \cdot y^{k-j} \cdot H_j(x) \cdot \tag{17}$$

After applying (17) in (15) we finally obtain a formula for our universal expansion coefficients:

$$\begin{gathered} b_{m,n} = \frac{1}{\gamma_m^{\alpha,\beta}} \times \\ \sum_{k=0}^{K} d_{k,n} \sum_{j=0}^{k} \binom{k}{j} H_j(h) g^{k-j} \int_{-1}^{1} \xi^{k-j} P_m^{\alpha,\beta}(\xi) w(\alpha,\beta,\xi) d\xi \end{gathered} . \tag{18}$$

The integral in (18) can be calculated analytically. The result of an integration depends on a relation of a Jacobi polynomial order with a value of k-j. The integral is 0 for k-j<m. When k-j=m the integral is:

$$I_m = \frac{\Gamma(\alpha+m+1)\Gamma(\beta+m+1)}{\Gamma(\alpha+\beta+2m+2)} 2^{\alpha+\beta+m+1}, \tag{19}$$

while for k-j>m the integral is given by:

$$I_{k-j} = I_m \binom{k-j}{m} {}_2F_1(m-k+j, \alpha+m+1, \alpha+\beta+2m+2, 2) \cdot \tag{20}$$

where ${}_2F_1(. , . , . , .)$ is a Gauss hypergeometric function [7].
In practice, for our diffraction example, the upper limit K in summation occurring in (18) varies from several up to thirty. The latter corresponds to the case when $0 \le \theta \le \pi$ and considered frequency range is 1GHz ≤ f ≤ 10GHz
It should be noted that when more than one variable is assumed to be stochastic at one time [1], the analogous procedure as the one presented through (1) – (20) hold true.

Finally having coefficients in (16) calculated for all frequency samples we can apply the vector fitting algorithm [8] in order to obtain the frequency dependent expansion coefficients in (18) in terms of rational functions [8, 9]. Afterwards the approximated transfer function (7) can be easily transformed into the time-domain by using the inverse Laplace transform. As a result we obtain the spectral expansion of an impulse response which is expressed by the sum of exponential functions what allows an application an effective calculation of a convolution with a given UWB signal.

The vector fitting algorithm is applied only once for each of the coefficient (16).

III. SIMULATION EXAMPLES

In this section we compare our universal polynomial chaos approach with standard Monte Carlo one in calculation of mean and standard deviation of exemplary $T(\omega_n,\xi)$ for frequency band 1–10 GHz with respect to a stochastic variable ξ following exemplary Beta distributions. As we pointed out in previous sections we present numerical examples for the case of a diffraction (creeping) ray transfer function [10]. The transfer function of a creeping ray contains a Fock type integral which was calculated by us numerically. We assume a stochastic behavior of θ while the radius of the cylinder is assumed to be known constant equal 0.25m. We present the results in Figs. 2-7. Each figure contains 4 graphs. The solid line corresponds to the results obtained with numerical calculation of (9). The dotted line relates to the results derived using (18) – (20). The square and circle symbol graphs correspond to Monte Carlo (MC) simulation results for different number of samples used. The graphs of mean and standard deviation characteristics are named with μ and σ, respectively. For space saving issues only real part of the functions is shown. In the first example θ has a Beta distribution with $\alpha=\beta=2$ in a range from c=0.01 rad to d=0.05 rad. Simulation results for this case are given in Figs. 2-3.

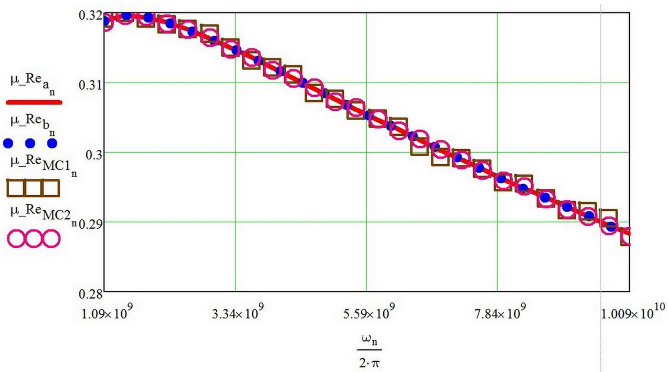

Fig. 2. Mean of a real part of an UTD creeping ray transfer function with respect to a frequency when θ has a Beta distribution with c=0.01 rad, d=0.05 rad, $\alpha=\beta=2$. MC results shown with squares and circles correspond to a number of samples 1000 and 10000, respectively.

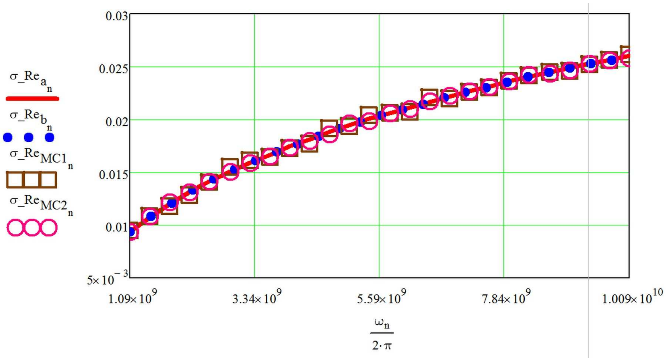

Fig. 3. Standard deviation of a real part of an UTD creeping ray transfer function with respect to a frequency when θ has a Beta distribution with c=0.01 rad, d=0.05 rad, $\alpha=\beta=2$. MC results shown with squares and circles correspond to a number of samples 1000 and 10000, respectively

In the second example we change a value of c into 0.4 rad and a value of d into 0.6 rad leaving the same values of α and β as in the first example. Simulation results for this case are given in Figs. 4-5.

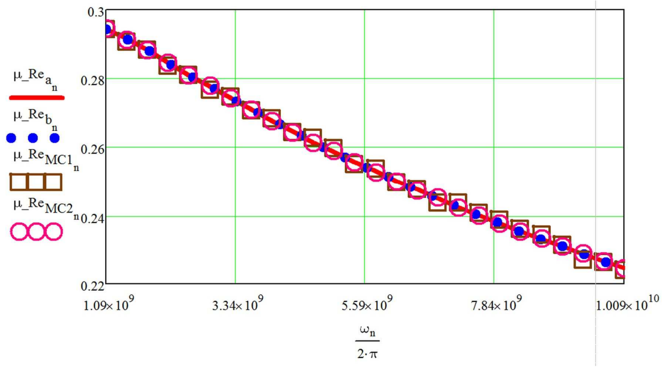

Fig. 4. Mean of a real part of an UTD creeping ray transfer function with respect to a frequency when θ has a Beta distribution with c=0.4 rad, d=0.6 rad, $\alpha=\beta=2$. MC results shown with squares and circles correspond to a number of samples 100 and 1000, respectively.

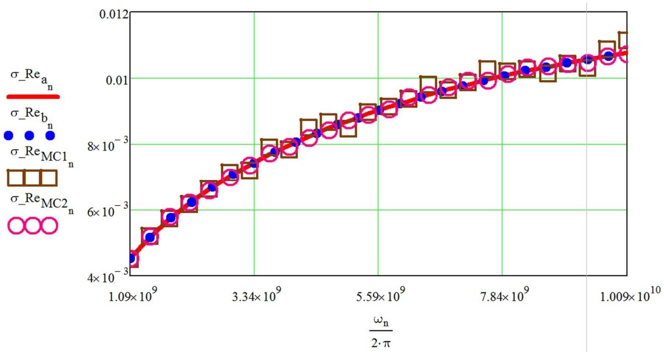

Fig. 5. Standard deviation of a real part of an UTD creeping ray transfer function with respect to a frequency when θ has a Beta distribution with c=0.4 rad, d=0.6 rad, $\alpha=\beta=2$. MC results shown with squares and circles correspond to a number of samples 1000 and 10000, respectively.

Finally we modify the shape of a Beta distribution by changing a value of α into 3 and a value of β into 7. Corresponding simulation results are shown in Figs. 6-7.

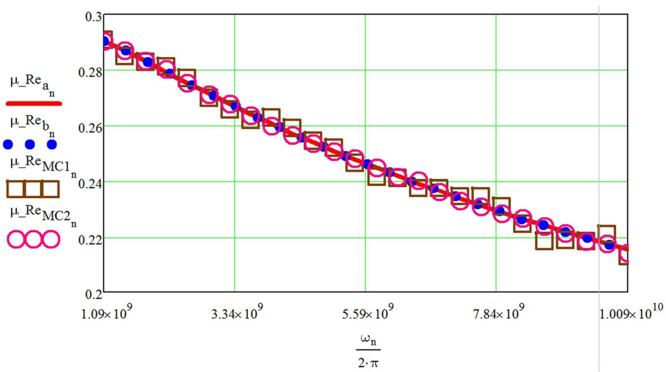

Fig. 6. Mean of a real part of an UTD creeping ray transfer function with respect to a frequency when θ has a Beta distribution with c=0.4 rad, d=0.6 rad, $\alpha=3$, $\beta=7$. MC results shown with squares and circles correspond to a number of samples 10 and 100, respectively.

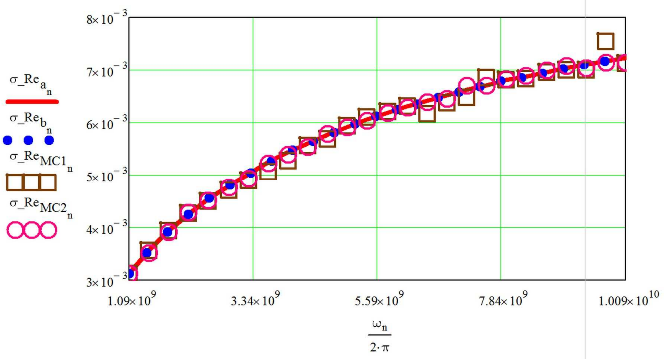

Fig. 7. Standard deviation of a real part of an UTD creeping ray transfer function with respect to a frequency when θ has a Beta distribution with c=0.4 rad, d=0.6 rad, α=3, β=7. MC results shown with squares and circles correspond to a number of samples 10 and 100, respectively.

When analyzing Figs. 2-7 we can see that all the results obtained with our universal approach are in a very good agreement with the results obtained using numerical calculation of (9) (solid lines). This numerical calculation of (9) was necessary for each of the examples. In our universal approach we do not need to apply any extra numerical calculations. The time efficiency of our approach is over 30 times higher than for the case of numerical calculation of (9) and over 500 times higher than for Monte Carlo method regarding to e.g. standard deviation results from Fig. 3, Fig. 5 and Fig. 7. Although we presented the simulation examples for the case of one convex obstacle the method can be easily adopted to the case of more obstacles in an UWB channel [9].

IV. CONCLUSIONS

We presented in the paper a new universal approach to polynomial chaos expansion for simulation of EM wave propagation in an UWB channel that focuses on statistical analysis of EM field distribution. We took advantage of an orthonormal basis of Jacobi polynomials which allows as to simulate propagation scenarios whose parameters has a Beta stochastic distribution which can model in particular a Gauss stochastic distribution and an uniform stochastic distribution. Parameters α and β of a Beta distribution allow to include in simulations variables with non-symmetric stochastic distributions. We presented and examined our new universal approach for the case of a diffraction phenomenon on a convex obstacle, which is a common element of an indoor propagation channel. The generality of our results express in validity of our expansion coefficients for wide range of possible values of a given propagation scenario variables without the need of performing any numerical calculations. The coefficient are the functions of all parameters of Beta distribution that is followed by a stochastic variable ξ. The coefficients (3) that are used to form the universal coefficients (18) need to be calculated only once for each frequency sample of a considered UWB spectrum and then can be tabulated. Application of vector fitting algorithm for approximation of (16) in a frequency-domain allows to obtain a very simple form (sum of exponential functions) of an impulse response corresponding to a given propagation phenomenon [9] what allows an application a very effective calculation of a convolution with a given UWB signal.

Our new approach enables to obtain stochastic distribution of a given transfer function in a very short time comparing to application of a Monte Carlo method while it allows flexible settings of parameters of stochastic variables in a wide range. The results obtained using our universal approach are very accurate what is presented for the case of exemplary stochastic variable distributions in Figs. 2-7.

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